Lesson 8: Fractions Greater than 1

Objective

By the end of the lesson, students will be able to label whole numbers as fractions (e.g., 2 and 8/4) and label points greater than 1 and between whole numbers as both mixed numbers and as fractions with the denominator greater than the numerator (e.g., 1 1/4 and 3/4).

What teachers should know...

About the math. A whole number can be represented as a fraction with a numerator equal to the denominator, and a fraction greater than 1 can be expressed as a fraction with a numerator greater than the denominator. In Figure A, the unit interval contains five subunits, and the arrow points to the sixth subunit from 0; this point on the line can be labeled as either 6/5 or the mixed number 1 1/5.

What number is the arrow pointing to?

Figure A

About student understanding. When students are asked to label fractions greater than one, they may focus on only some of the relevant information on the line. In Figure B, a student counted all 6 tick marks for both denominator and numerator, and labeled the tick mark 6/6, without considering the marked unit. In Figure C, a student counted the 5 subunits within the unit interval to determine the denominator of 5, but then counted only the one subunit beyond 1 when determining the numerator, and labeled the tick mark 1 1/5.

Figure B

Figure C

About the pedagogy. This lesson introduces two ideas: whole numbers as fractions (1 = 4/4, 2 = 8/4), and numbers at the same place on the line have the same value, a supplement to the order principle. To investigate, the class creates a human number line as illustrated in Figure D. Cards are labeled with values from 0 to 2. Values include: whole numbers (0, 1, 2), fractions (1/4, 2/4, 3/4), whole numbers as fractions (0/4, 4/4, 8/4), mixed numbers (1 1/4, 1 2/4, 1 3/4), and fractions greater than 1 (3/4, 9/4, 7/4). Students match these number cards to “human tick marks” on a “race course,” and discover that some of the human tick marks are holding more than one number card! The class discusses the idea that a point on a line can be labeled with different forms; for example, the whole number 2 can be labeled as the fraction 8/4, and the mixed number 1 1/4 can be labeled as 5/4. Reasoning about these ideas requires revisiting the definitions for denominator, numerator, and fraction.

Figure D
Common Patterns of Partial Understanding in this Lesson

Counting tick marks (or intervals) without considering unit-subunit relationships
- There are six tick marks after 0, and the arrow is pointing to the sixth one, so it's 6/6.

What number is the arrow pointing to?

A. \(\frac{6}{6}\)  B. \(\frac{6}{5}\)  C. \(\frac{1}{5}\)

Determining numerator based on distance from a number other than 0
- It's one fifth after 1, so it's \(\frac{1}{5}\).

What number is the arrow pointing to?

A. \(\frac{6}{6}\)  B. \(\frac{6}{5}\)  C. \(\frac{1}{5}\)

Assuming there can be only one name for a point on the line
- I figured out it's \(1\frac{1}{5}\), so I wrote it in because the answer wasn't shown.
Lesson 8 - Outline and Materials

Lesson Pacing

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Total time: **55 minutes**

Materials

Teacher:
- Transparency C-rods
- Transparency markers
- Transparencies:
  - Opening Discussion Transparency
- Whiteboard C-rods
- Magnetized yardstick
- Dry erase markers
- Principles & Definitions Poster -- Integers (section for Order)
- Principles & Definitions Poster -- Fractions (section for Whole Numbers as Fractions)
- Materials for human number line activity:
  - Piece of rope to measure between the human tick marks (3’ to 5’)
  - Number cards, cut out

Students:
- Worksheets

You will select 9 students to represent human tick marks on a “race course.” The rest of the class will label the human tick marks with the numbers on cards:
- whole numbers
- fractions
- mixed numbers
- fractions greater than 1
- whole numbers as fractions
Lesson 8 - Teacher Planning Page

- Numbers at the same place on the line have the same value.
- A whole number can be written as a fraction with a numerator equal to the denominator.
- A fraction that is greater than 1 can be written as a mixed number or as a fraction with the numerator greater than the denominator.

Objective

By the end of the lesson, students will be able to label whole numbers as fractions (e.g., 2 and \( \frac{8}{4} \)) and label points greater than 1 and between whole numbers as both mixed numbers and as fractions with the denominator greater than the numerator (e.g., \( 1\frac{1}{4} \) and \( \frac{5}{4} \)).

Useful questions in this lesson:
- What rod is the unit? What rod is the subunit?
- How can we use the rods to figure out the denominator? the numerator?
- Can we label this distance from 0 with more than one number? Why?
Opening Problems 5 Min

The opening problems introduce the idea that a number greater than one can be expressed as a fraction with a numerator greater than the denominator.

Yesterday we worked with mixed numbers like $1\frac{1}{4}$, and today you’ll learn another way to name fractions greater than one.

It’s okay if the Opening Problem is challenging. We’ll learn about fractions greater than 1 in the lesson.

Rove and observe the range in students’ ideas.

This problem engages students in:

- labeling a fraction greater than 1 on a line with the unit and subunits marked
Opening Discussion 15 Min

1. Debrief: Identify a fraction greater than 1
2. Add to Order and record Whole Number as Fractions

- Numbers at the same place on the line have the same value.
- A whole number can be written as a fraction with a numerator equal to the denominator.
- A fraction that is greater than 1 can be written as a mixed number or as a fraction with the numerator greater than the denominator.

1. Debrief: Identify a fraction greater than 1

Review definitions for denominator, numerator, & fraction, and stress relationships between unit and subunits.

Which number is the arrow pointing to on the line?

If you chose 6/6, can you explain your thinking?

They started at 0 and counted six tick marks, and decided it’s 6/6. But they forgot about the unit interval - there are 5 subunits in the unit, not 6.

I counted six spaces, so it’s 6 sixths.

What about 6/5?

There are 5 subunits in the unit, so the denominator is 5. The 1 is the same as 5/5 and the arrow is at pointing to one more fifth, 6/5.

If you label the tick marks from 0, it goes 1/5 2/5 3/5 4/5 5/5 6/5.

What about 1/6?

They knew the subunits were fifths, but they forgot that the number is greater than 1 because it’s to the right of 1. They forgot the order principle.

That was my answer. There are 5 fifths in the unit, and the arrow is pointing to another fifth.

Talk to a partner: What do you think the answer is now?

Take a vote.

The answer is 6/5, and let’s use yellow and white rods to figure out why.
Let’s first figure out the denominator.

- What rod can we use to show the unit interval?
- What rod can we use to show the subunits in the unit interval?
- Now, what is the denominator? How do you know?
  - The denominator is 5, because there are 5 subunits in the unit.
- What is the numerator for each tick mark? Use the definition for numerator.

Guide students to label:  \( \frac{1}{5} \), \( \frac{2}{5} \), \( \frac{3}{5} \), \( \frac{4}{5} \)

Can we continue and label 1 as \( \frac{5}{5} \)?
  - Yes, because 5 fifths fill up the unit interval.
  - No, \( \frac{5}{5} \) isn’t a real fraction!
  - No, 1 is a regular number, not a fraction.

Record:

\[
\frac{5}{5} = 1
\]

The rods show that \( \frac{5}{5} \) and 1 are different labels for the same point. So \( \frac{5}{5} \) and 1 have the same value on the line.

Discuss relationship between the numerator and denominator.

When you look at \( \frac{5}{5} \), what do you notice about the numerator in relation to the denominator?
  - The numerator is equal to the denominator!
  - The denominator is the number of subunits in the unit, and there are 5 subunits. The numerator is the number of subunits from 0, and that’s also 5.

Discuss 0.

Does 0 also have another name?
  - Yes, it’s zero fifths.
  - No, 0 can’t be a fraction.

The definition of numerator is “the number of subunits from 0.” Well, 0 is 0 fifths from 0, so we can label it \( \frac{0}{5} \).
Add a white rod, and discuss the value of the point indicated by the arrow

**How many fifths is the arrow from 0?**
- Green circle: It’s one more fifth than 5 fifths, so it’s 6 fifths. That’s answer B.
- Yellow circle: No, it has to be 1 1/5, but that answer isn’t listed.

**The arrow is pointing to one more fifth beyond 1. The pattern is 0/5 1/5 2/5 3/5 4/5 5/5 6/5, and the arrow is at 6 fifths.**

### Pushing Student Thinking:
Assuming there can be only one name for a point on the line

**We decided on 6/5, but shouldn’t the answer be 1 1/5? Talk to a partner.**
- Green circle: 1 1/5 is equal to 6/5 -- both are one more fifth than 1.
- Green circle: Both have the same subunits - fifths. They both have 6 fifths.
- Yellow circle: Only 6/5 is correct because it’s a fraction, and we’re studying fractions!

Record:

\[
\frac{6}{5} = 1\frac{1}{5}
\]

6/5 and 1 1/5 are different labels for the same point. 5 fifths equal the length of the unit, and 6 fifths is one more fifth than 1.

Discuss relationship between the numerator and denominator.

**When you look at 6/5, what do you notice about the numerator in relation to the denominator?**
- Green circle: The numerator is greater than the denominator.
- Green circle: The denominator is the number of subunits in the unit, and the numerator is the number of subunits from 0. There are 5 subunits in the unit, and the arrow is 6 subunits from 0.

**So we can write a fraction greater than 1 as a mixed number or as a fraction where the numerator is greater than the denominator.**
2. Add to **Order** Identify Whole Number as Fractions

Point to the **Order** principle on the Integers poster.

The order principle tells us that numbers increase from left to right, and decrease from right to left. We’ve never talked about numbers at the same place on the line have the same value - numbers like 1 = 5/5, and 1 1/5 = 6/5.

Let’s add that numbers at the same place have the same value.

Record addendum on Integers poster.

Point to the line for Mixed Number on the Fractions poster.

We’ve been talking about new kinds of numbers. We figured out that we can label whole numbers with a fraction, like 1 = 5/5. Let’s record this idea.

On this number line, what do the numbers 0, 1, 2, 3, 4 tell us?

- Those are whole numbers.
- They mark the unit intervals.

How many subunits are in each unit, and what do we call these subunits?

- There are three subunits in each unit, so they’re thirds.
- The denominator is 3.

The denominator is 3, and we call the subunits “thirds.” If we write fractions for each of these whole numbers, what are the numerators?

- How many thirds do we have at 0?
- in the distance from 0 to 1?
- in the distance from 0 to 2?

- There are no subunits for 0, so it’s 0/3.
- There are 3 subunits for 1, so it’s 3/3. And 6 subunits for 2, so it’s 6/3. For 3, it’s 9/3.
- It’s skip-counting by 3s! 3 subunits is 1 unit, 6 subunits is 2 units, 9 subunits is 3, like that.

Record the values \[ \frac{0}{3}, \frac{3}{3}, \frac{6}{3}, \frac{9}{3}, \frac{12}{3}. \]

The point for 0 has two labels, 0 and 0/3, and the point for 1 has two labels, 1 and
$\frac{3}{3}$, and the same for the other whole numbers. Let’s call this principle “Whole numbers as fractions.”

So 1 and $\frac{3}{3}$, are at the same place on the line, and have the same value. And 2 and $\frac{6}{3}$, are at the same place on this line, and have the same value.

Let’s explore this idea with a class game.
Class Activity

This ‘human number line’ activity highlights relationships among whole numbers, fractions less than 1, mixed numbers, and fractions greater than 1.

1. Create a “human number line”
2. Students label the human tick marks with number cards.
3. Record the human number line on whiteboard.

1. Create a “human number line”

Today we’ll make a “human number line” in the classroom and label the tick marks on the line. Some of you will be “human tick marks,” and some of you will label the tick marks with number cards.

Hold up cards. Select 9 students to be the human tick marks on the number line.

The human tick marks will use this rope to space themselves out evenly to create a number line. Let’s imagine that the rope is a subunit of $\frac{1}{4}$ mile, and you have to stand exactly 1 subunit or $\frac{1}{4}$ mile apart. Measure belly button to belly button, like this!

Demonstrate:

- The first two students hold the rope to their belly buttons and move apart until the rope is straight.
- This pattern is continued until all 9 students are spaced evenly.

Our human tick marks have to make sure they stay in their own spot!
2. Students label the tick marks with number cards.

Other students will label the tick marks.

Distribute the number cards:
- Whole Numbers
- Fractions
- Mixed Numbers
- Fractions Greater than 1
- Whole Numbers as Fractions

Our number line starts at 0, and the tick marks are \( \frac{1}{4} \) mile apart. When you label a tick mark, you both have to agree that the number is a correct label.

(a) Label 0 and \( \frac{0}{4} \)

Who has 0? Place 0 on the human number line.

These prompts support student reasoning:
- Labelers, how did you decide where to label 0? What principles helped you?
- Tick Mark, how did you decide whether to accept 0?

The student who has \( \frac{0}{4} \) will ask to place his/her number also!

- I have \( \frac{0}{4} \), and that’s also correct! \( \frac{0}{4} \) is another name for 0.
- It’s the new principle, “Whole numbers as fractions.”

Review **whole numbers as fractions** to support students’ understanding that 0 = \( \frac{0}{4} \).

0 is a whole number, and it can also be expressed as a fraction.

(b) Label \( \frac{1}{4} \), \( \frac{2}{4} \), \( \frac{3}{4} \)

Who has a fraction for our next human tick mark?

These prompts support student reasoning:
- Labeler, how did you decide that your number belongs at this tick mark?
- Tick Mark, do you accept this number? How did you decide?

Yes, it’s \( \frac{1}{4} \), because each subunit is a fourth, and I’m one fourth from 0.

What about the next tick mark?

Repeat to place \( \frac{2}{4} \) and \( \frac{3}{4} \).
(c) Label 1 and \( \frac{4}{4} \)

What number is the next tick mark after \( \frac{3}{4} \) on this human number line?
- 1, and I have that number!
- \( \frac{4}{4} \), and I have that number!

Tick Mark, can you be 1 and \( \frac{4}{4} \)?
Why is the numerator equal to the denominator?

Have students explain using **whole numbers as fractions**.

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**Pushing Student Thinking:**
Assuming a fraction must be less than a whole number

Another student said that, if she ran \( \frac{4}{4} \) mile, it would be less than running 1 mile, because \( \frac{4}{4} \) is a fraction. What do you think?

- If you divide a mile into four equal subunits, and you run all four of them, then you run a whole mile.
- \( \frac{4}{4} \) is a fraction, and fractions are less than 1.

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(d) Place mixed numbers:  \( 1 \frac{1}{4} \),  \( 1 \frac{2}{4} \),  \( 1 \frac{3}{4} \)

Who has the card showing a mixed number after 1 on our number line?

- \( 1\frac{1}{4} \).
- 2.

These prompts support student reasoning:
- Labeler, how did you decide that your number belongs at this tick mark?
- Tick Mark, do you accept this number? How did you decide?

What mixed number is after \( 1\frac{1}{4} \)?

Repeat to place \( 1\frac{2}{4} \) and \( 1\frac{3}{4} \).
(e) Place fractions greater than 1: \(\frac{5}{4}, \frac{6}{4}\) and \(\frac{7}{4}\)

Now let’s find places for \(\frac{5}{4}, \frac{6}{4}\) and \(\frac{7}{4}\). Where can you put those numbers? Most of the Tick Marks are already labeled!

These prompts support student reasoning:

- Labeler, how did you decide that your number belongs at this tick mark?
- Tick Mark, do you accept this number? How did you decide?

After all mixed numbers are labeled, ask:

If I run from 0 to the 1\(\frac{1}{4}\) mile marker, have I run the same distance if I run from 0 to the \(\frac{5}{4}\) marker? Talk with a partner.

Yes, because there are 4 fourths in 1, and another fourth makes 5 fourths.

I’m not sure.

Have students role play running from 0 to 1\(\frac{1}{4}\) (5/4) to count the subunits and verify that they have run 5 fourths of a mile.

We can label the Tick Marks greater than one with either a fraction or a mixed number. 1\(\frac{1}{4}\) is equivalent to \(\frac{5}{4}\) because both are 5 subunits from 0.

(f) Place 2 and \(\frac{8}{4}\)

So what numbers are left?

I have 2! and I have \(\frac{8}{4}\)!

This prompt supports student reasoning:

If I run 2 miles, have I run \(\frac{8}{4}\) miles?

Yes, the new principle is whole numbers as fractions. One mile is four fourths of a mile, and the next mile is another four fourths of a mile, and that’s eight fourths all together.

Tick Marks should remain standing so you can record the line on whiteboard.
3. **Record human number line on board.**

Let's record our human number line before our Tick Marks sit down!

Record the human number line. Use one of the longer rods (such as yellow) so it’s easy to see the subunits.

Do not erase this number line. You will refer to it in the Closing Discussion.
Use the recorded number line on the board to review the big ideas.

1. Fractions as distances from 0 on the line
2. **Whole Number as Fractions**
3. Equivalence of mixed numbers and fractions greater than one

- Numbers at the same place on the line have the same value.
- A whole number can be written as a fraction with a numerator equal to the denominator.
- A fraction that is greater than 1 can be written as a mixed number or as a fraction with the numerator greater than the denominator.

### 1. Fractions as distances from 0 on the line

Encourage students to apply *unit*, *subunit*, *denominator*, and *numerator*. Also discuss numerator-denominator relationships.

[Student] was at 3/4 of a mile. How can we prove that place was 3/4 mile?

These prompts support student reasoning:

- **Where is the unit interval, and what are the subunits?**
  - The unit interval is from 0 to 1. There are four subunits, or fourths.
- **How do we know this point is 3/4 of a mile?**
  - The denominator 4 is the number of subunits. The numerator is the number of subunits from 0, and I count 3 subunits.
- **Why is the numerator less than the denominator?**
  - 3/4 is less than 4/4 or 1.

### 2. Whole Numbers as Fractions

Encourage students to apply *unit*, *subunit*, *whole number as fractions*, and the revised definition for *order*. Also discuss numerator-denominator relationships.

[Student] was holding two number cards. How can we prove that [student] could stand at 1 mile and fourth fourths of a mile?

These prompts support student reasoning:

- **Where is the unit interval on this line, and what are the subunits?**
  - The unit interval is from 0 to 1. There are four subunits, or fourths.
- **How do we know both 1 and 4/4 are the same distance from 0?**
  - The two numbers are the same distance from 0, like it says in the order principle: Numbers at the same place have the same value.
  - The two numbers have the same number of subunits. There are four fourths in 1 mile.
- **Why is the numerator equal to the denominator?**
  - Because the unit was divided into fourths, and 1 is the distance of four fourths from 0.
3. Equivalence of mixed numbers and fractions greater than one

[Student] was holding two number cards. What principles prove that [student] could stand at 1²/₄ mile and ⁶/₄ of a mile?

These prompts support student reasoning:

- **Where is the unit interval on this line, and what are the subunits?**
  - The unit interval is from 0 to 1. There are four subunits, or fourths.
- **How do we know both 1²/₄ and ⁶/₄ are the same distance from 0?**
  - There are six fourths in 1²/₄ because there are four fourths in 1, and two more is six fourths.
  - The numerator is greater than the denominator, because [student] was further than 1 mile.
- **Why is the numerator greater than the denominator?**
  - Because the unit was divided into fourths, and 1 is the distance of four fourths from 0.
Students complete the closing problems independently.

The Closing Problems are an opportunity to show what you’ve learned about whole numbers as fractions and about two ways to label fractions greater than one.

If you’re still confused, I’ll work with you after the lesson.

This problem assesses how students:

- label a fraction greater than 1 on a line with the unit and subunits marked

Collect and review to identify students’ needs for instructional follow-up.
Fractions Lesson 8: Fractions greater than 1

On this page, write a mixed number and an improper fraction for the number the arrow is pointing to.

Example:

0 \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{1}{3} \quad 3

Mixed number \quad \frac{1}{3} \quad Fraction greater than 1 \quad \frac{2}{3}

1. \quad \begin{array}{c}
0 \\
1 \\
2 \\
3 \\
4 \\
\end{array}

Mixed number \quad \frac{3}{4} \quad Fraction greater than 1 \quad \frac{2}{3}

2. \quad \begin{array}{c}
0 \\
1 \\
2 \\
\end{array}

Mixed number \quad \frac{7}{2} \quad Fraction greater than 1 \quad \frac{5}{2}

3. \quad \begin{array}{c}
0 \\
1 \\
2 \\
\end{array}

Mixed number \quad \frac{1}{2} \quad Fraction greater than 1 \quad \frac{3}{2}

Fractions Lesson 5: Fractions greater than 1

On this page, write the numbers where they belong on the number line.

4. Write the following numbers where they belong on the number line below.

\begin{array}{cccccccc}
\frac{3}{4} & 0 & \frac{5}{4} & \frac{6}{4} & \frac{10}{4} & 1 & \frac{2}{4} & 2 \frac{1}{4} \\
\end{array}

\begin{array}{cccccccc}
\frac{1}{4} & 0 & \frac{5}{4} & \frac{6}{4} & \frac{10}{4} & 1 & \frac{2}{4} & 2 \frac{1}{4} \\
\end{array}

5. Write the following numbers where they belong on the number line below.

\begin{array}{cccccccc}
\frac{2}{5} & \frac{17}{5} & \frac{6}{5} & 2 & \frac{5}{5} & 1 & \frac{1}{5} & \frac{6}{5} \\
\end{array}

\begin{array}{cccccccc}
\frac{5}{5} & \frac{2}{5} & \frac{5}{5} & \frac{1}{5} & 1 & \frac{1}{5} & \frac{6}{5} \\
\end{array}