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3 Emergent Mathematical Environments in Children's Games

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In their daily lives, children engage in a wide range of cultural practices: They play games, participate in sports, and sell goods. For many, such activities are the staples of childhood, and much of what has come to be called out-of-school mathematics learning occurs through children's participation in such practices.

What kinds of environments emerge in everyday cultural practices in which children engage in mathematics learning? The question presents difficult problems for analysis. We know that learning environments are not presented to individuals and thus directly observable by analysts; rather, they are constructed by individuals in activity. Further, such constructions are deeply interwoven with historical achievements as well as cultural values and norms. Much of the existing research on children's learning environments have not produced adequate coordinations of these epistemological and cultural issues in descriptions of learning environments in practices.

Sociological approaches often skirt efforts to analyze practices entirely. Instead, such approaches often use distal features of children's environments (social class, economic organization), properties of individuals (race, gender), or small-group organization in classrooms (e.g., reward structures) to analyze effects of these "proxy" variables on children's achievements. Although such distal variables as social class or reward structure may be predictive of children's accomplishments (e.g., Johnson, Johnson, & Stanne, 1985; Slavin, 1980, 1983), such approaches can provide little insight into the learning environments that emerge in children's practices.

Social-interactional approaches have offered useful fine-grained analyses of children's turn-for-turn exchanges in terms of categories of questions and expla-

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nations (Webb, 1982, 1991) or social conflicts (Ames & Murray, 1982; Botvin & Murray, 1975; Doise & Mugny, 1984) and how these variables affect children's achievement. However, we do not find critical analyses of mathematics in these analyses or the way in which aspects of culture are interwoven with individuals' mathematical environments.

Ethnographic approaches often point to cultural aspects of children's activities as ingredient to children's learning environments (Clark, 1983; Eckert, 1989). Although such approaches are more sensitive to the cultural features of practices, they do not offer frameworks for the systematic analysis of mathematical environments nor how such environments might come to be represented in individuals' activities.

In this chapter, we describe a framework for the analysis of children's learning environments in which a core construct is children's *emergent goals* (Saxe, 1991). To illustrate the approach, we focus on our recent work on children's play of a game in which children become engaged with mathematical problems.

SOME CORE ASSUMPTIONS OF THE EMERGENT GOALS FRAMEWORK

Central to our work is the view that an understanding of the mathematical environments that emerge in the game requires the coordination of two analytic perspectives (Saxe, 1991). The first is a constructivist treatment of children's mathematics: We take as a core assumption that children's mathematical environments cannot be understood apart from children's own cognizing activities (Piaget, 1952, 1977; Saxe, 1991; Steffe, von Glasersfeld, Richards, & Cobb, 1983; von Glasersfeld, 1992). Indeed, mathematical environments take form as children construct and accomplish goals and subgoals that are grounded in their prior understandings. Such goals may be relatively elementary, such as those that a child constructs in counting a collection of objects, or relatively sophisticated, such as those that an adolescent constructs in creating a geometrical proof. Regardless, mathematical environments become constituted only as individuals structure mathematical goals.

The second perspective derives from sociocultural treatments of cognition (e.g., Laboratory of Comparative Human Cognition, 1986; Rogoff, 1990; Saxe, 1991). Children's construction of mathematical goals and subgoals is interwoven with the socially organized activities in which they are participants; whether computing batting averages or making change for lemonade, children construct goals that are framed by cultural artifacts (e.g., currency or number systems), activity structures (e.g., the rules and objectives of playing Monopoly), and social interactions.

The game that is the focus of our analysis was designed specifically to favor the emergence of particular kinds of mathematical goals in children's peer interactions.[1] Through analyses of videotapes of children's play, our aim was to provide some insight into children's emergent mathematical environments.

TREASURE HUNT AND THE EMERGENT GOALS FRAMEWORK

The game of Treasure Hunt is depicted in Fig. 3.1. To play the game, children assume the roles of treasure hunters in search of "gold doubloons," gold-painted base-10 blocks in denominations of 1, 10, 100, and 1000. In play, children collect their gold in treasure chests that consist of long rectangular cards organized into thousands, hundreds, tens, and ones columns, and children report their quantity of gold on their gold register with the number orthography. The child who acquires the most gold wins the game.

The game has a turn-taking structure. Children take turns rolling a die on a large rectangular playing board that consists of six islands (see Fig. 3.2). A roll shows how many islands a player hops until landing. Each island contains a trading post where a player may purchase supplies and four geographical regions where players receive messages that offer opportunities either to use their supplies to gain additional gold or to protect their existing gold. An enlargement of Snake Island—its trading posts and its geographical regions—is given in Fig. 3.3. At the end of each turn, players must report the quantities of their gold in their treasure chest on their gold register; players are subject to "challenges" from their opponents for inaccurate reports (e.g., reporting 9 hundreds, 8 tens, and 15 ones [9(100) + 8(10) + 15(1)] as "9815"[2]).

Emergent Goals in the Play of Treasure Hunt

A basic assumption of the Emergent Goals Framework is that goals are not fixed or static constructions, but rather take form and shift as children participate in practices. In this process, goals are necessarily interwoven with cognitive and

[1]Members of the peer interaction research group at UCLA participated in the development and/or analysis of the game. These individuals included Joseph Becker, Teresita Bermudez, Kristin Droege, Tine Falk, Steven Guberman, Marta Laupa, Scott Lewis, Anne McDonald, David Niemi, Mary Note, Pamela Paduano, Laura Romo, Geoffrey Saxe, Rachelle Seelinger, and Christine Starczak.

[2]Henceforth, we use the following notation to indicate quantities of blocks: We indicate in parentheses the denominational value of a block; for instance, a block of value 100 is denoted as (100), and a block of value 10 is denoted as (10). We indicate the number of blocks of a specified denomination by an integer to the left of the block denomination. Thus, 9 blocks of value 10 are denoted by the expression, "9(10)".

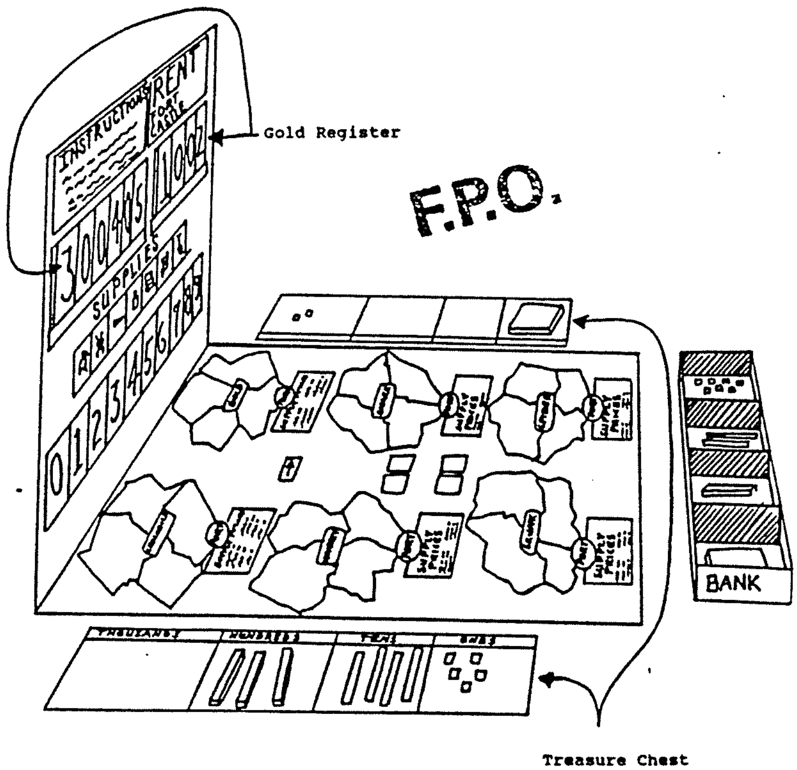

FIG. 3.1. The Treasure Hunt game.

socio-cultural aspects of children's functioning. The Emergent Goals Framework targets four principal parameters (see Fig. 3.4): activity structures, social interactions, artifacts/conventions, and prior understandings. Next, we sketch these parameters, pointing to the way in which they serve as a frame for an analysis of emergent goals in Treasure Hunt.

Parameter 1: Activity Structures

In our analyses of the activity structure (Parameter 1) of Treasure Hunt, we distinguish between an *intended structure* and an *actual structure*. The intended structure consists of the rules, objectives, and organization of play as prescribed by the designers of Treasure Hunt. The actual structure, in contrast, is the game that emerges as children play. Each serves an important function in our analyses: Our specification of the intended structure provides a schema of the organization of play as presented to children, whereas our analyses of the actual structure provides a description of the transformation of this structure in the play of children. In our empirical analyses of play, a central concern is with the way the

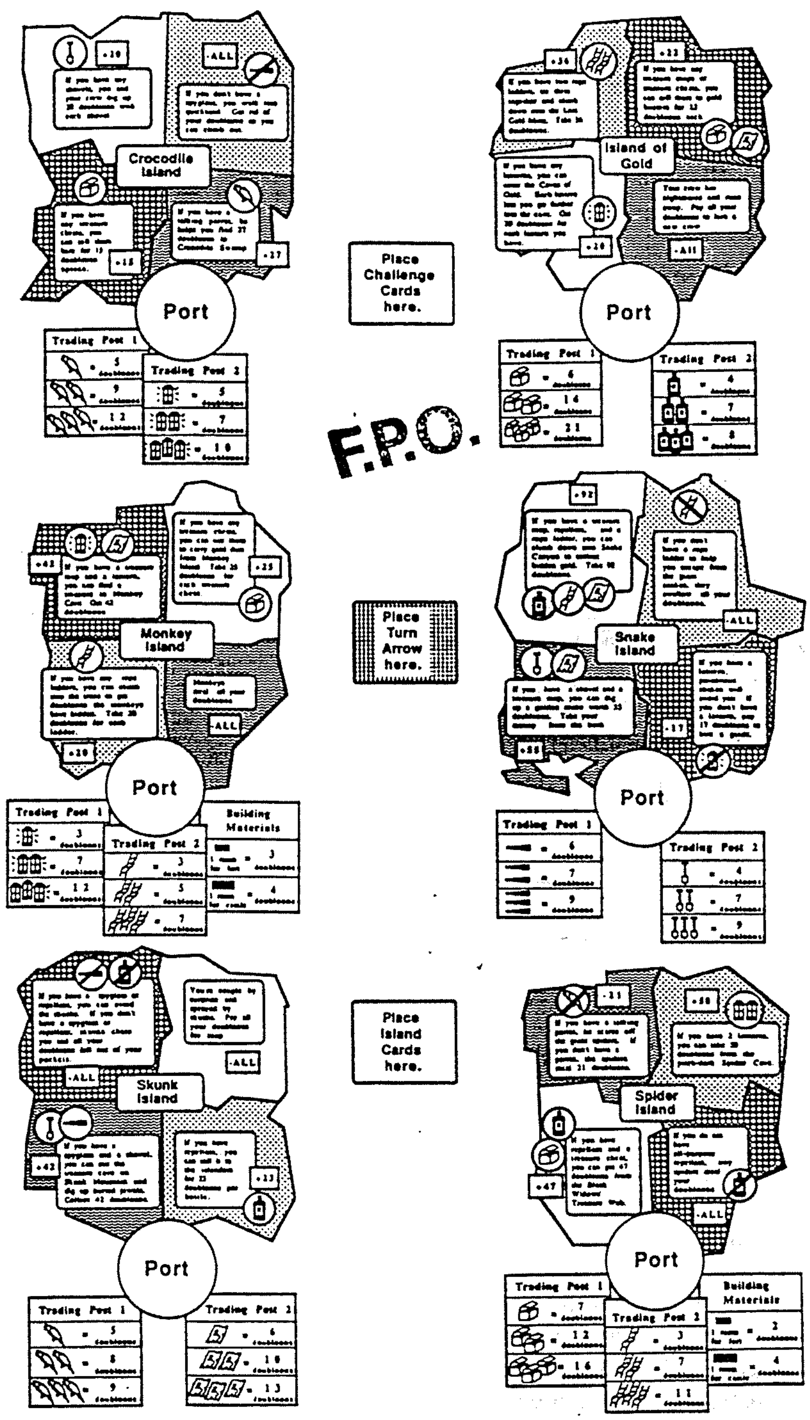

FIG. 3.2. The six islands on the playing board in Treasure Hunt.

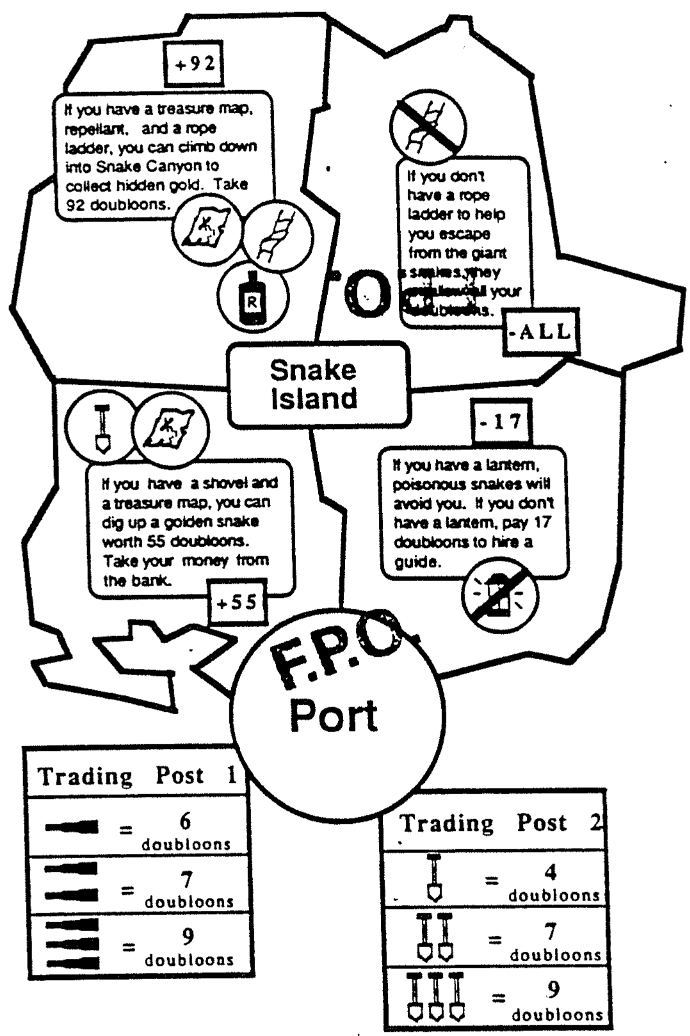

FIG. 3.3. An enlargement of Snake Island.

actual structure of play—the emergent rules by which children play, the values that they form in play, and their own particular routines by which they play—is interwoven with children's emergent mathematical goals.

The Intended Structure. Our prescribed objective for players of Treasure Hunt is to acquire gold, and the rules of the game specify a routine turn-taking organization. Each player begins the game with a specified quantity of gold (players generally started with 9 hundreds, 5 tens, and 6 ones blocks [9(100),

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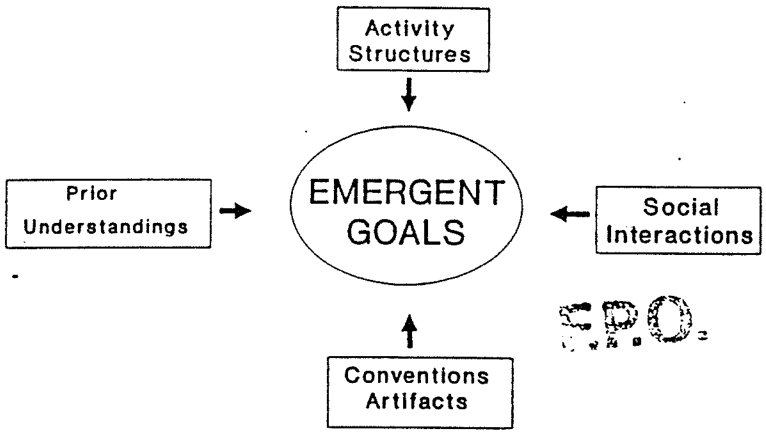

FIG. 3.4. The four parameters of the Emergent Goals model.

5(10), and 6(1)]). This quantity is both contained in each player's treasure chest and represented in numeric form on each player's gold register. Play begins when the first player rolls the die and moves his or her ship to one of the six islands as a function of the roll.

After moving to the appropriate island, a player's turn consists of an ordered sequence of five routine phases (see Fig. 3.5). In the first phase, the *challenge*, the player has the option of questioning whether the opponent's numeric repre-

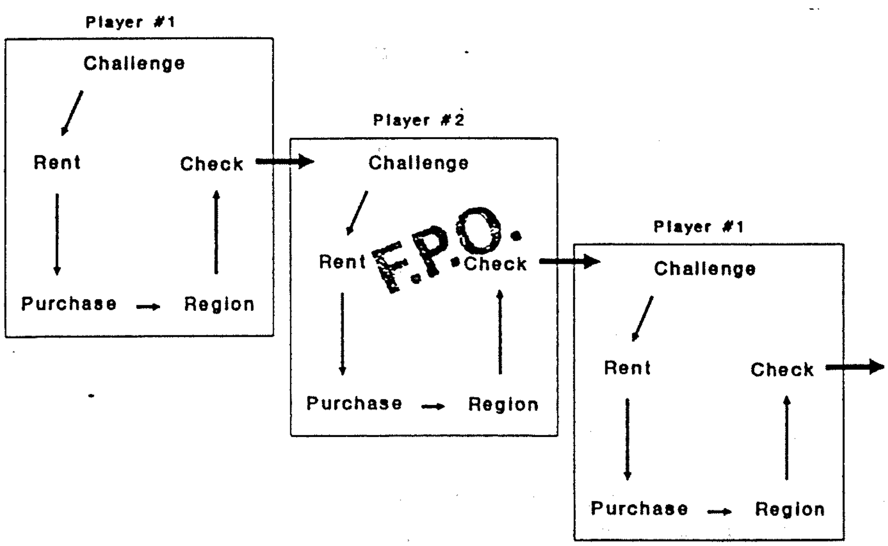

FIG. 3.5. The five-phase turn-taking structure of Treasure Hunt.

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sentation in the opponent's gold register actually reflects the appropriate quantity of gold in the opponent's treasure chest. To initiate a challenge, the player draws a challenge card from the center of the playing board. The card contains a number, indicating how many doubloons the player will receive from the bank if the player's challenge to the opponent is, in fact, appropriate (as determined by player–opponent negotiation). The second phase of a turn, the *rent*, occurs if the player lands on an island on which the opponent has placed a fort or castle (previously purchased and positioned by the opponent). If so, the player is obligated to pay the opponent a specified amount of gold. The *purchase* phase follows; the player now has the option of purchasing supplies using the two or three menus contained at the port of the player's island (see Fig. 3.3). Next, the player draws a colored card that initiates the *region* phase, in which the player moves—as a function of the color on the card—to one of four colored regions of the island. At the colored region, the player receives a printed message indicating whether the player may trade some of the specified supplies to either gain gold or to avoid losing gold (see Fig. 3.3). Finally, after a purchase is complete, the player, in a *check* phase, adjusts the gold register (numeric representation of quantity) to adequately reflect the amount of gold in the treasure chest (see Fig. 3.1). Once the phase is completed, the player turns an arrow on the center of the game board toward her opponent, signaling that the turn is completed.

The intended structure of Treasure Hunt has various implications for children's emergent mathematical goals in play. In the purchase phase, for instance, players should buy supplies at island trading posts, attempting to add or multiply supply values and then subtract the sum from their gold, and perhaps even attempt to accomplish price-ratio comparisons. In the region phase, players draw island cards that send them to particular island areas where, depending on the particular region, they must add gold to their chests in exchange for certain supplies, or they must pay for gold if they lack certain supplies by subtracting a value of gold from their treasure chest. Later, in the check phase, children compare their gold and gold registers to make sure that their gold registers (orthographic representations) adequately represent their quantities of gold (base-10 block representations); this cross-representation comparison goal is supported by their opponents' license to challenge, the phase that begins the opponent's turn. Thus, the intended structure of the game—the objective to acquire gold, the rules of play, and the cyclical phase organization—was designed to support the emergence of various kinds of mathematical goals.

Actual Structure. The actual structure of play is a principal target of empirical analysis. In play, children transform the intended structure—an external definition of how to play—into their own rules, values, and routines. Although the intended rules are presented as external prescriptions, the actual rules are the ones that define for them what is legitimate and important in play. Similarly,

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while the intended objectives (e.g., to acquire more gold) are defined externally for children, in play children form their own values that guide their own objectives. Finally, although we present a routine structure of play (e.g., our five phases), children in the course of play form their own idiosyncratic routines. Thus, the intended structure of play defines a potential organization of play that is realized in different ways by children in activity. Children's emergent mathematical goals take form in relation to this emergent activity structure.

We sketch next three additional parameters, each of which is necessary for the analysis of children's emergent mathematical goals. These consist of the artifacts and conventions that may become interwoven with children's mathematical activities, the social interactions during play that may either constrain or enable children's construction of mathematical goals, and the prior understandings that form the cognitive basis of children's construction of mathematical goals.

Parameter 2: Artifacts, Conventions

There are several artifacts and conventions (Parameter 2) that are intrinsic to play that influence the character of children's mathematical goals. These include the price-ratio menus, base-10 blocks (gold doubloons, Fig. 3.6), and the numerals for representing gold. During play, children's mathematical goals are interwoven with properties of these artifacts. Consider, for instance, an arithmetical problem that may emerge in the purchase of supplies and the implications for accomplishing the purchase using two different sets of artifacts. First, the player must sum the prices of the specified number of supplies, keeping tallies of prices and number of supplies as prescribed by the different price ratios (a form of addition linked to the price ratios). Then, in the purchase of supplies, a player needs to accomplish a subtraction problem; the child's goals will differ as a function of whether the child calculates using the base-10 blocks or using the standard orthography. For instance, to perform the subtraction in gold doubloons, the player may generate goals and subgoals involving equivalence trades of larger blocks for smaller blocks in order to accomplish the subtraction; in

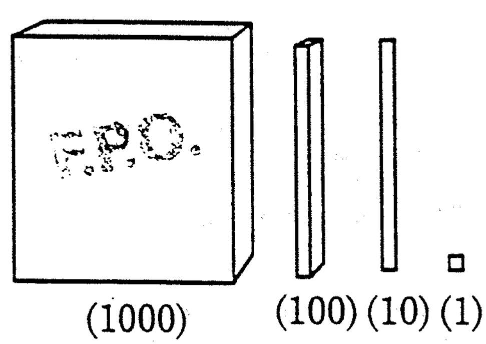

FIG. 3.6. Exemplars of the four denominations of base-10 blocks used as gold doubloons.

contrast, with the orthography, the player may apply the school-linked column subtraction procedure with borrowing.

Parameter 3: Prior Understanding

The prior understandings (Parameter 3) children bring to Treasure Hunt have implications for the mathematical goals that emerge in play. For Treasure Hunt, prior understandings may include children's knowledge of board games as well as their knowledge of basic arithmetic operations. For instance, some children have difficulty understanding the denominational structure of the blocks. They may treat all blocks with a value of unity, not conceptualizing blocks with reference to their many-to-one equivalence relations [e.g., 10(1) is equivalent to 1(10)]. As a result, when faced with a problem that requires payments when one does not have exact change to pay [e.g., paying 14 when one has only 8(100) 1(10)], a child will structure different kinds of subgoals in the formation and accomplishment of the arithmetical problem. Thus, goals are rooted in children's conceptual constructions, and analyses of processes of goal formation must be grounded in a treatment of children's understandings.

Parameter 4: Social Interactions

Children's goals often shift and take form as individuals participate in practice-linked social interactions (Parameter 4). For instance, in the purchase of supplies that cost 14 doubloons without exact change, a child who has difficulty accomplishing the payment may receive assistance with the more difficult aspects of the problem from his or her opponent. Such assistance may have the effect of reducing the complexity of the arithmetical goals that the child structures and accomplishes in the problem. Rather than conceptualizing the trade of a large for middle doubloon values, the trade may be accomplished by the opponent, and the player may merely need to form and accomplish goals of paying the exact amount (adding single or multidenominational doubloon pieces).

The Emergent Goals Framework and Children's Mathematical Environments

The view put forth here is that the mathematical environment known by the child is no more than the process of mathematical goal and subgoal formation and accomplishment. From this perspective, the Emergent Goals model provides a basis for the analysis of the mathematical environments children structure in practices. In analyzing aspects of the mathematical environments that emerge in Treasure Hunt, we guide our analyses by the three principal constructs that define the actual structure of play—children's rules, values, and routines.

Rules

Rules are warrants that are used to define what is and what is not legitimate in play. In this section, we point to the way rules, an aspect of the actual activity structure of play, are interwoven with children's emergent mathematical goals and how rules themselves may emerge and take new forms in the course of play.

In the design of Treasure Hunt, we developed a rule structure that we believed would involve students in sustained play and at the same time lead them to structure some rich mathematical environments. Recall that rules of play were linked to each phase of the game. For instance, in the purchase phase, children could buy supplies using prices from the menus of the island that they had landed on. They could buy as many supplies as they liked, drawing gold from their own treasure chests and depositing the payment in the bank. In actual play, some children embellished the rules, others simplified the rules, and still others played more or less faithfully by them. In the interactions discussed next, we focus on some of the emergent purchase-phase rules and the way these rules were interwoven with the emergence of children's mathematical environments.

Like most children, in their play Monica and Jackie frequently made use of base-10 blocks, the principal artifact of play (Parameter 2). We know from observations of their play and prior assessments that Monica and Jackie had difficulty understanding denominational transformations [like 1 one hundred block is equivalent to 10 ten blocks, 1(100) = 10(10) (Parameter 3)]: Whenever they were "running low" on ones and tens doubloon pieces in supply purchases, they merely drew a challenge card and then collected from the bank the amount indicated on the card.[3] Through their invented rule, Monica and Jackie, through a negotiated process (Parameter 4), created a means of preempting the emergence of base-10 block problem that would require them to make an equivalence transformation [e.g., trade 1(10) for 10(1) or 1(100) for 10(10)]. Now, they merely counted single-unit blocks (or a combination of single- and multiunit blocks) to pay for a supply purchase. Thus, the mathematical environment for these children emerged as adding units, or multiples of units, to produce a particular sum.

Toni and Veronica's play, described next, presents an interesting instance in which children's emergent rules led them to construct more complex goals. In the prescribed rules of play, children were allowed to buy supplies sold only on the island on which they had landed, a rule that Toni and Veronica chose to ignore early in their first session of the game:

> Toni landed on Skunk Island and indicated to Veronica that she wanted to purchase spyglasses. Indicating that no spyglasses were sold on the island, Veronica then proceeded to look at other island supply menus to determine on which island they were sold (with Toni's tacit agreement that this was a legitimate activity.) Veronica located a spyglass menu on Snake Island, and quoted Toni the price (6 doubloons for 1 spyglass), whereupon Toni paid and took the supplies.

[3]As indicated earlier, the intended use of these cards was that they be drawn when players questioned their opponents report (in numerals) of their gold (base-10 blocks).

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In this interaction, Veronica and Toni reached an implicit agreement about a new rule: You can buy any supply regardless of where it is sold. The new rule led to Toni's formulation of a subtraction goal—to pay 6 from her treasure chest. Thus, the new rule (Parameter 1) in conjunction with the interaction between players (Parameter 4) framed the emergence of the problem. Further, the representational embodiment of the goal was set in the context of the base-10 blocks, a principal artifact of play (Parameter 2), and the cognitions entailed in formulating and accomplishing the goal necessarily involved an understanding of base-10 block arithmetical transformations (Parameter 3).

Values

Children came to value some aspects of the game over others. Children's emergent game-linked values were central in their creation of their own objectives of the game. Like emergent rules, we found that children's values varied and that they could enhance as well as limit the complexity of children's goals. We next consider two emergent values during play: a value linked to obtaining a "best buy" in the purchase of supplies, and a value linked to acquiring the "thousands" block.

Finding a "Best Buy." Consider an interaction during a purchase phase in Toni and Veronica's last session of play.[4] Toni constructs the goal to compare ratios and the appropriate subgoals that allow her to accomplish this comparison successfully:

> Toni says: "I wanna buy two chests and two . . . " [points to the ladders and says] "and that's it, 'cause two of these over here" [Spider Island] "is seven and two over here" [Monkey Island] "is five, so I'll get them over there."

During her turn, Toni compared the price for two ladders at two different islands, and decided not to buy them where she had landed because they cost more. Toni formulated the goal of ratio comparison: two for seven doubloons versus two for five doubloons, reaching the conclusion that two ladders for seven doubloons is a more expensive price than two for five doubloons. Then during Veronica's turn, Toni advised Veronica what to purchase based upon her comparison of price ratios, further evidence that she has come to value best buys.

It is instructive to compare this example in which Toni guides her activity by her value to find a "best buy" with the prior example in which Toni and Veronica formed a new implicit rule. Toni and Veronica's earlier rule-linked interaction consisted of an across-island search for supplies. The search occurred because

[4]After 2½ months of playing, they were no longer playing by the new rule they had created during the first session but were abiding by the prescribed rules.

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Toni wanted to find spyglasses (a single supply). The across-island search and the discovery that the same supplies were sold for different values on different islands could well have provided a context for Toni's construction of a new value—obtaining a "best buy." The value led her to form ratio comparison goals, conceptually more complex goals than other children formed when they were making purchases.

The 1000 Piece. A more common value than obtaining a "best buy" was that of acquiring enough doubloons to obtain a 1000-doubloon piece, the largest denomination used in the game. In observing Jorge play with Felix, we found that Jorge came to value obtaining a 1000 block and that his value resulted in more complex mathematical goals.

Prior to showing evidence that he regarded the 1000 block in any special way, Jorge had not formed goals that involved equivalence transformations of his blocks, despite sanctions we built into the game to encourage doubloon equivalence trades.[5] Indeed, Jorge had left the gold in noncanonical form—more than 9 units of a single denomination [e.g., 12(1) instead of 1(10) + 2(1) pieces]. With the emergence of the concern to get the 1000 block (a new value), Jorge formed goals of trading across denominations in order to obtain the valued block. In the following excerpt, we observe an instance of the import of the 1000 block for Jorge's emergent goals.

> Jorge has just purchased two parrots and two lanterns and then draws an orange region card (if you have a parrot, collect 27 doubloons). He looks through his supplies and finds one parrot. His opponent gives him 2(10)s and 7(1)s, and Jorge now has 9(100)s, 8(10)s and 20(1). He once again expresses a desire of "getting one like that" (referring to the 1000). He counts his ones up to ten, then ten more. Then says, with sudden realization, "Twenty, twenty. I change, I change, I wanna change!" He trades, saying: "I change for a thousand."

As the excerpt shows, Jorge became involved in a trading problem that involved forming and accomplishing several consecutive many-to-one correspondence goals. When he counted his 20 ones, he realized that it would result in his total of 1000: We infer that he calculated a series of trades—20 ones could be traded for 2 tens; those 2 tens would be added to his already existing 8 tens to form a total of 10 tens; those 10 tens could be traded in for an additional hundred block, which together with his existing 9 hundreds would total 10 hundreds; in

[5]Some messages on the geographical regions required that children deposit in the bank all of their one-doubloon pieces (see, e.g., the upper right region on Snake Island in Figure 3.3). This requirement was designed to encourage children to trade ones pieces for tens pieces, so as to minimize possible losses. For example, having 11(1) as opposed to 1(1) + 1(1) would subject a child to risk of losing 11 doubloons as opposed to a risk of losing only 1 doubloon.

turn, those 10 hundreds could be traded for the 1000 block. Jorge did not physically accomplish every trading step; he was able to mentally work out the equivalence of 9 hundreds, 8 tens, and 20 ones, to 1000.

In a subsequent game, we noted Jorge's interest in the 1000 block leading to new kinds of arithmetical goals. Rather than simply trading to determine whether he could obtain a 1000 block, Jorge became concerned with anticipating through additions and subtractions how much gold he would have to add to his treasure chest in order to obtain a 1000 block. For instance, at one point Jorge had 9(100) + 8(10) + 7(1), and he stated to his opponent that he needed two more tens to have 1000. Later in play, Jorge had 9(100) + 9(10) + 3(1) and stated the he needed "one more [ten]."

In contrast to Jorge and his opponent, Ralph's play illustrates how valuing the 1000 block could limit the complexity of emergent mathematical goals. On one occasion, we observed that Ralph's doubloons approached 1000 (he had 993). In order to keep his doubloons, Ralph reduced his supply purchases, since every payment for supplies would further deplete his treasure chest, taking him further from his targeted 1000 block. As a result, Ralph reduced the complexity and frequency of emergent mathematical goals. Unfortunately, there was some irony to his strategy. Over the course of the game, Ralph never did acquire the appropriate supplies that would allow him to garner additional gold to obtain his targeted block.

In the examples just cited related to finding best buys and obtaining the 1000 block, we find that the mathematical environments that emerged in play are constituted by children's emergent mathematical goals and interpretable from the perspective of the Emergent Goals model. In all cases, children's values (Parameter 1) were the basis for structuring problems in which mathematical goals emerged, whether they involved ratio comparisons or block trades and computations. These goals were shaped by the principal artifacts of play (Parameter 2), price ratios, and base-10 blocks, and the goals took form in the context of the give-and-take exchanges with their opponents (Parameter 4). Finally, both the ratio comparison goals and the computations involving the thousand block necessarily involved at least an incipient understanding of the mathematics involved in these concepts (Parameter 3).

Routines

Children's idiosyncratic routines were organizing schemas that, like rules and values, had implications for the emergence of children's mathematical goals. Common classes of routines were related to purchasing and to challenging. The purchase routines varied from regular purchases of large quantities of supplies to regular purchases of only one or two items for exact change. Sometimes purchase habits became shared by players, usually by mutual imitation, and thus we found large-scale purchasers playing with one another (and, reciprocally, small-scale purchasers playing with one another). Challenging routines included challenging after every change in gold and regular warnings by opponents of possible upcoming challenges. We next consider the way both kinds of routines were related to children's emerging mathematical goals.

Routines Involving Purchases. The routine of purchasing multiple supplies was shared by Ramiro and David. This routine led both players to structure arithmetical problems of addition, subtraction, and multiplication (successive additions), equivalence trades, and translations of base-10 blocks into the standard number orthography. The following excerpt illustrates well the complexity of goals that can emerge when children routinely buy large quantities of supplies.

> On his turn, Ramiro lands at the port on Monkey Island, reads the price menus, and decides to buy all of the castle rooms available—19—at a cost of 4 doubloons each. He pays with one one hundred piece and takes change, trades to put his gold in canonical form, and then changes his gold register. Next, he draws a colored island card which sends him to Monkey Island's white region. The crux of the region message reads, ". . . if you have a ladder, collect 20 doubloons." Being well-stocked with supplies, Ramiro presents 3 ladders, takes 6 tens from the bank and uses an additional 4 tens in his treasure chest to trade for a 100 doubloon piece, changing his gold register, accordingly, to 900.

As a part of his routine purchase of large quantities of supplies (buying 19 castle rooms at 4 doubloons each), Ramiro led himself down a path in which he came to structure the goal of adding 4, 19 times. During payment, he then formed a complex subtraction goal—to subtract 86 from his 916 in gold. After collecting change, he formed and accomplished two additional mathematical goals: He transformed his gold into canonical form by trading 10(1) for 1(10), and then represented his gold in the standard orthography in his gold register. During his turns, David engaged in similar purchasing behavior, buying, for example, all of the available fort rooms. Although some of their calculations were incorrect, in this process of mathematical goal formation and accomplishment they were constructing complex arithmetical environments.

Like Ramiro and David, Fanny and Carla also shared a purchasing routine in their play. However, unlike David and Ramiro, their routine led to emergent goals of quite a different order. Fanny and Carla would buy only those supplies for which they had exact change; further, when they did make a purchase, they would only buy one kind of supply at a time. An excerpt provides a typical example of this type of transaction routine:

> Fanny had 9(100) 4(10) 1(1) in her treasure chest when she found herself at the at the port on Monkey Island. The cost of items at the port were three, four, five, seven, and twelve doubloons. She could not pay any of those amounts with exact change, and chose not to purchase anything.

The routine of purchasing only when exact change was available clearly limited the emergence of addition and subtraction goals because it reduced the instances in which Fanny and Carla could purchase and pay.

Routine and Nonroutine Challengers. All children were told about rules linked to challenges—that they could challenge only at the beginning of their turn, and that they needed to change their gold register at the end of their turn to avoid being challenged by their opponent. Despite children's knowledge of the challenge rules, many children routinely declined to challenge. Others, however, developed some interesting routines linked to challenging that led to the emergence of particular kinds of mathematical goals.

Jose and Guni played together and developed a shared routine. After Guni challenged Jose a few times early in the game, both of these players came to challenge each other with great regularity. To defend against one another's challenges, both players did not wait until the end of their respective turns to change their register. Instead, both changed their registers whenever the amount of gold was altered in their treasure chests.

Due to their routine and frequent challenges, both players came to form goals linked to translating the base-10 block representations of gold doubloons into the standard number orthography quite regularly in their play. Perhaps due to the routine challenges, Jose and Guni showed more efficient means of maintaining accurate correspondences: Sometimes they performed the math (adding or subtracting) directly on the gold register. In such cases, they changed their gold registers prior to their gold transactions, representing in the orthography the anticipated outcome of their forthcoming gold transaction.

In the examples cited involving purchase and challenge routines, we find mathematical environments constituted by children's emergent mathematical goals, goals linked to each of the four parameters. Children's routines (Parameter 1) were the basis for structuring the complexity levels of arithmetical and representational problems in purchases and base-10 block translations into the standard orthography. These goals were shaped by the principal artifacts of play (Parameter 2), base-10 blocks, and the standard number orthography, and the goals took form in the context of the sometimes heated interactions of challenging their opponent or buying from their opponent (Parameter 4). Finally, the arithmetical and representation goals necessarily involve at least an incipient understanding of the mathematics involved in these activities (Parameter 3).

CONCLUDING REMARKS

In our analyses of Treasure Hunt, we found that despite the fact that children were ostensibly playing the same game, using the same materials, and participating in the same classrooms, children were often engaged with different mathematical environments. Indeed, in our analyses of emergent goals we found that the structure of Treasure Hunt emerged over play, with children structuring varying rules, values, and routines. We found that sometimes children's emergent rules, values, and routines served to limit the potential complexity of the mathematical environments children structured in play, as when children adopted a new purchase-phase rule that allowed them to circumvent the construction and accomplishment of goals entailed in subtraction problems involving trades; at other times, children's rules, values, and routines enhanced the mathematical complexity, as when a child came to value best buys, leading to incipient ratio comparison goals. Although rules, values, and routines were central constructs for understanding the dynamic activity structure of play, to capture the character of children's emergent mathematical goals required us to anchor our analyses in the other three parameters of the Emergent Goals model. Indeed, whether we considered the emergent goals linked to best buys or to simplifying subtraction problems with trades, children's mathematical goals could not be well understood without coextensive analyses of the artifacts in play (base-10 blocks, numerals, price ratios), the mathematical understandings children brought to the game (e.g., their understanding of part–whole relations in denominational structures), and emergent social interactional processes (e.g., conflicts, negotiations, agreements).

In closing, the issues that we confronted in our analyses are fundamental ones for research addressed to the representation of social and cultural processes in children's mathematics. Children's learning environments, whether in or out of school, can only be adequately understood insofar as we can document the goals with which children are engaged. Understanding how particular practices support and limit children's goal-directed activities is both a critical feature of sociocultural analyses of children's learning and key to the design and modification of educational practices.

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