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STUDYING MATHEMATICS LEARNING IN COLLECTIVE ACTIVITY

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Abstract

This paper describes a study of the interplay between social and developmental processes in children's mathematics learning. The focus is on children's play of an educational game, Treasure Hunt, and the way children's interactions in play frame developmental processes involving arithmetic with base-10 blocks. Sixty-four third and fourth graders were grouped in same- and mixed-grade dyads. Analyses of interactions revealed that players were frequently involved with jointly structuring arithmetical problems involving base-10 blocks. However, the arithmetical goals that members of dyads created often differed as labor became divided in their activity. Two findings were of particular interest: (1) differences in divisions of labor as a function of players' grades and grades of their opponents led to construction of different arithmetical goals, and (2) differences in goals led to different sequences in children's strategic developments, sequences that differed from the developmental trajectory in our matched controls. © 1998 Elsevier Science Ltd. All rights reserved.

Studying Mathematics Learning in Collective Activity

Increasingly, students of learning problematize the relation between individual and context in cognitive analyses. At the crux of the issue is the observation that solutions to cognitive tasks do not reside "in the head" of individuals; rather, cognitive activities are "stretched over" artifacts and people as individuals draw on social and material resources to conceptualize and accomplish everyday tasks (cf. Bateson, 1972; Cole, 1991, 1996; Forman & Larreamendy-Joerns, 1995; Goodwin & Duranti, 1992; Lave, 1988; Olson, 1976; Rogoff, 1995). This means that whether we look to the "best buy" calculations in the grocery shopping of middle class housewives (Lave, 1988) or the accomplishment of family errands by children in Brazilian shantytowns (Guberman, 1996), task solutions are not reducible to strictly internal mental processes. Rather, they are intrinsically linked to social

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interactions and other dimensions of collective practices that both constrain and enable individuals' accomplishment of everyday tasks (Saxe, 1991; Saxe *et al.*, 1996).

Like others concerned with contextual analyses of development, we argue that there is much to be gained for our understanding of learning by problematizing individual-context relations, though this analytic turn has its pitfalls. Perhaps our greatest concern is that the focus on distributed aspects of cognition in "sociocultural" treatments is often at the neglect of a coordinated analysis of the cognitive activities of individuals. In this short paper, we sketch an approach that is our effort to coordinate cognitive analyses of group and individual in collective practices, with a particular focus on children's mathematics. We focus our remarks on children's joint play of an educational game that we developed, Treasure Hunt, with the belief that the analytic tack we present here is applicable to understanding children's mathematics learning in collective activities across a broad range of cultural practices.*

The Analytic Approach

We begin with two key assumptions about cognition.

The first assumption is central to most constructivist formulations of cognitive development. Children create the possibility for developing new mathematical understandings through their efforts to structure and accomplish the mathematical goals that emerge in their activities—goals that are linked to individuals' prior understandings and the problems that emerge in their practices. It follows from this assumption that if we are to understand children's learning in practices, we need to develop methods that offer insight into the mathematical goals that children are structuring and accomplishing as well as children's prior understandings that support these goals (see Saxe, 1991).

The second assumption is concerned with the relation between individuals' goals and collective activities. We assume that, as participants in a collective practice, individuals' goal-directed activities are constitutive of group efforts to accomplish problems; reflexively, individuals' goal-directed activities take form in relation to the work of self and other towards problem accomplishment. This means that if we are to understand children's learning in activity, we need to develop methods for the analysis of practice that provide insight into the interplay between (a) the mathematical goals that particular individuals are structuring and (b) the problems that the collective is accomplishing.

Our work on Treasure Hunt flows from these two assumptions. Below, we present analyses of both the collective cognitive work children produce in collective play and the learning that takes form as children individually create and accomplish emergent goals.

The Play of Treasure Hunt

The Treasure Hunt game is depicted in Fig. 1. To play, pairs of children assume the roles of treasure hunters in search of "gold doubloons," gold-colored base-10 blocks in

*More detailed analyses of the analytic approach and results that we sketch here are contained in Saxe & Guberman (1998).

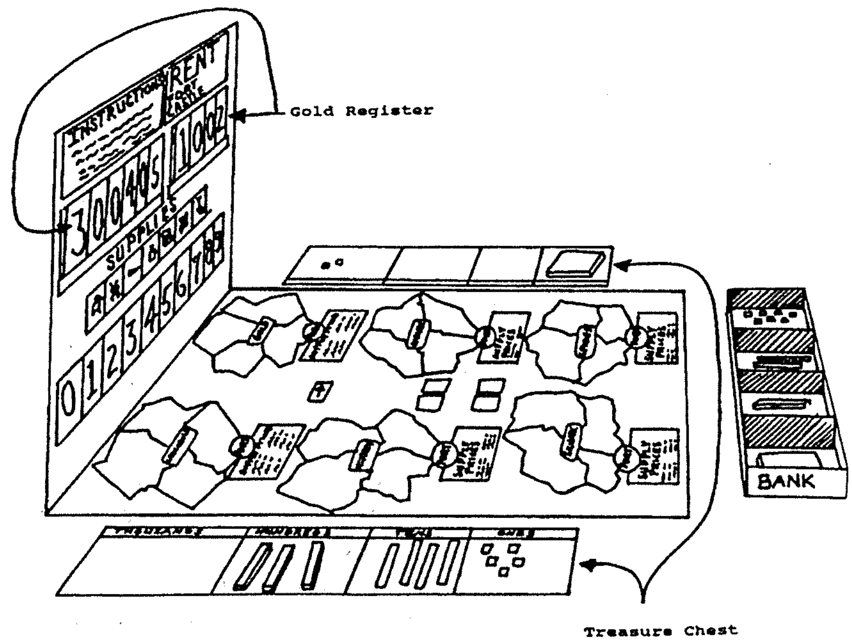

Figure 1. Treasure Hunt.

denominations of 1, 10, 100, and 1000 units. Children collect their doubloons in "Treasure Chests," rectangular cards organized into thousands, hundreds, tens, and ones columns, and report the quantity of their gold with numerals on the "Gold Register." The child who acquires the most gold wins the game. At the start of each turn, a child moves his or her toy ship around the board as a function of a die roll. In the course of the game, children frequently purchase supplies (e.g., talking parrots, lanterns) with their gold, report their value of gold on their Gold Register (with printed numerals), and forfeit and gain doubloons by drawing special message cards (e.g., "If you have a lantern, poison snakes will avoid you. If you don't have a lantern, pay 17 doubloons to hire a guide."). In this process, children become engaged in producing, translating, and performing arithmetic on quantitative representations in the media of base-10 blocks, spoken language, and number orthography (numerals).

Our Study of Learning in Play

To study the relation between children's joint activity in Treasure Hunt and the mathematical goals that individual players were structuring during that activity, we videotaped pairs of third and fourth grade children (8- to 10-year-olds) in their classrooms at an urban elementary school in Los Angeles. The math achievement scores for children in the school were low. The median score on the California Test of Basic Skills for both third and fourth graders at the school in comparison to national (US) norms was in the lowest quartile, and the scores of children who participated in the study were representative of the school population.

For our project, we targeted dyads at play who were of similar and mixed ability levels in math. Both dyadic configurations are common in children's collective activities in schools today.

Selection of children was accomplished in several steps. All third and fourth grade children whose parents had signed consent forms were administered the Wide Range Achievement Test (WRAT) in arithmetic. From this population, 96 students (48 third graders and 48 fourth graders) were assigned to one of three groups. Group assignment was accomplished through a stratified procedure based upon WRAT scores, grade level, and classroom.

To create dyads composed of children of *similar* ability, we paired students within grade level: third graders were paired with third graders, and fourth graders were paired with fourth graders. Further, we made sure that no children within a given grade who scored very high on the WRAT (the top 15% of scorers) were paired with children who scored very low on the WRAT (the bottom 15% of scorers).

To create dyads composed of children of *mixed* ability, we paired third graders and fourth graders. In this pairing, we made sure not to assign third graders who achieved high WRAT scores with fourth graders who achieved low WRAT scores.

Once the pairing procedure was accomplished for both similar and mixed dyadic groups, we discussed with teachers the pairings, making minor adjustments based upon teachers' anticipation of difficulties (e.g., personality conflicts). In the end, 32 children were assigned to a group of mixed-ability dyads (16 dyads of third graders playing fourth graders), and 32 children were assigned to a control group (16 third graders and 16 fourth graders).* An equal number of children from each participating classroom were assigned to each of the three experimental groups.

To conduct the twice weekly $\frac{1}{2}$-hour Treasure Hunt play sessions, children shifted classrooms. One classroom was devoted solely to similar ability dyadic play. Another classroom was devoted solely to mixed-ability dyadic play. The control children were assigned to non-Treasure Hunt classrooms.

Children played Treasure Hunt for about a 10-week period. Children's play was videotaped twice: once when they were less expert at play (their first play session after receiving instruction) and once when they were more expert (their last session). The game supported the children's regular math curricula, which included multi-digit arithmetic. During Treasure Hunt play time, the matched non-players were engaged with various math and non-math linked activities during the class time the players were engaged with Treasure Hunt. Through post-test tasks we assessed both players' and non-players' understandings of key mathematical knowledge.

Largely due to the transient population of families in the school, our sample was reduced in size over the course of the experimental period. In the end, 38 third graders completed the 10-week project of whom 10 were assigned to the similar ability group, 14 to the mixed-ability group, and 14 to the control group. Forty-two fourth graders completed the 10-week project; of these 12 were assigned to the similar ability group, 14 to the mixed-ability group, and 16 to the control group. A few additional adjustments in dyad pairing

*While similar ability and mixed ability groups were playing the targeted game, the control group participated in other math enrichment activities of the teachers' own choice in separate classrooms. Over the course of the 10-week period, these activities ranged from activities with manipulatives consisting of arithmetical problem solving to whole class activities involving arithmetic worksheets.

were made over the course of the 10-week project period. All adjustments that were made respected the original principles for the creation of dyads.

Emergent Representational and Arithmetical Problems in Play

One type of arithmetic problem that occurs frequently in the play of Treasure Hunt emerges in supply purchases. A player* who lands on Snake Island (Fig. 2), for example, may select to purchase supplies such as two shovels and two spyglasses. As indicated in the supply price menu in Fig. 2, the price to the player would be 14 doubloons if the

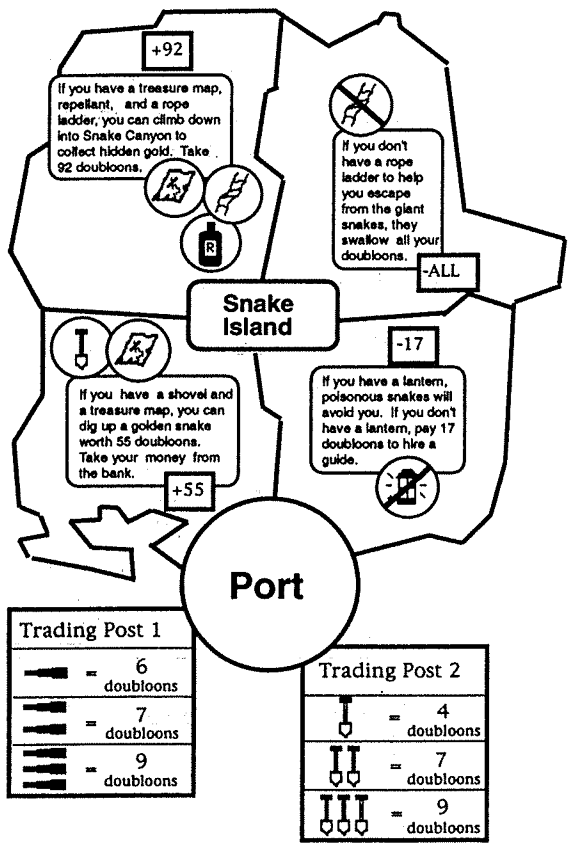

Figure 2. Snake Island, one of six islands on the Treasure Hunt game board.

*For each dyad, we refer to one member as the "player" and one member as the "opponent." These two roles change with each turn. A child is referred to as the "player" when it is his or her turn. The child is referred to as the "opponent" when it is not his or her turn.

shovels and spyglasses were purchased in couplets. One of two types of arithmetical problems can issue from this decision (see Fig. 3).

First, if the player has an ample distribution of ones and tens doubloons, the ensuing problem may be relatively straightforward, entailing an exact payment of 14 1-pieces or one 10-piece and four 1-pieces (depicted in Fig. 3, top). We term this a "direct payment" problem and the solution an "exact payment."

Second, if the player does not have a distribution of doubloons to produce an exact payment, the player has two principal options to solve the "indirect payment" problem. The player may use a "trade–pay" strategy—an extension of the direct payment strategy; in this case, the player trades one 10-piece for ten 1-pieces and then produces an exact payment of 14 doubloons (see Fig. 3, middle). Alternatively, the player may employ a "remove–replace" strategy; in this case, the player removes two 10-pieces from the Treasure Chest (putting them in the bank), replacing them with six 1-pieces (Fig. 3, bottom). Although the remove–replace strategy accomplishes the equivalent of the trade–pay strategy, its mathematical rationale is more opaque since the exact payment of 14 doubloons never occurs. (In some sense, a trade is "hidden" in the remove–replace strategy.) In the flow of play, direct and indirect payment problems become the focus of joint activity, though individuals often construct and accomplish quite separate goals in their distributed solutions.

Analyses of Players' Solutions to Indirect Payment Problems in Collective Activity

In order to analyze children's problem solving and mathematics learning in joint play, we identified every time direct and indirect payment problems emerged. For the purpose

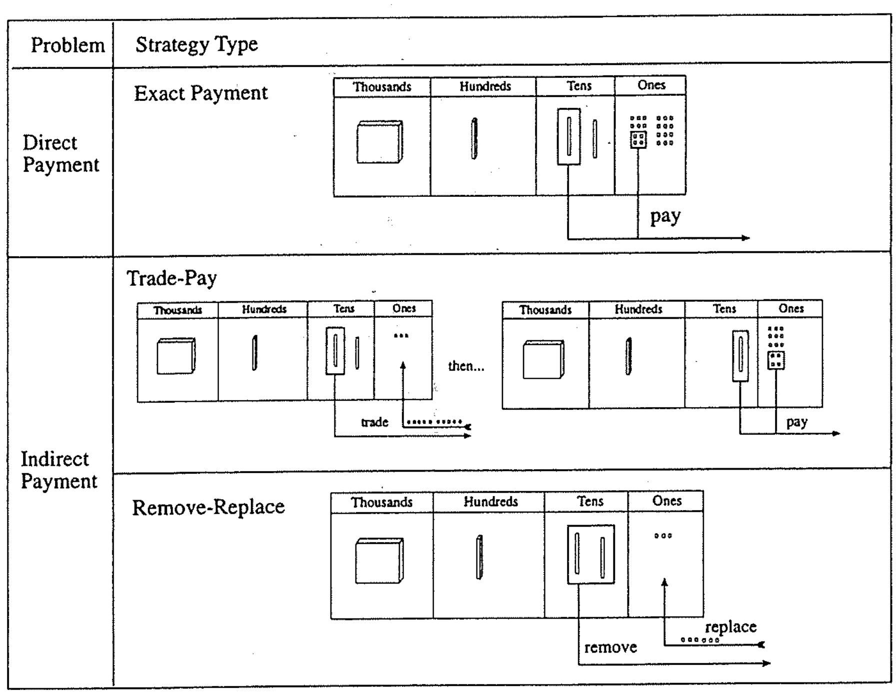

Figure 3. Problem and strategy types for paying 14 doubloons.

of this paper, we focus only on indirect payment problems. We found that players' solutions to indirect payment problems were distributed through two principal forms of social interaction.

In the case of *direct assistance*, the opponent provides greater or lesser levels of assistance to the player's efforts to complete a supply purchase. This assistance may be in the form of helping to produce a count of doubloons or a trade of greater denominations for lesser denominations. Regardless, in the accomplishment of the supply purchase problem, players' numerical goals take form and are modified as a function of the activity of the opponent. Typically, such solutions involved trade–pay strategies.

In the case of *thematically organized assistance*, mathematical goals take form in relation to players' efforts to accomplish joint problems that arise from roles many children invented in play: in such instances, children assume the roles of customer and storekeeper in accomplishing purchases. These roles were not prescribed or even suggested in the game rules but were created by many children in their play, presumably through drawing on their everyday experiences in commercial activities and their play of other games. In thematically organized interactions, the player becomes the customer, typically selecting supplies to purchase and producing an overpayment (sometimes with the assistance of the storekeeper). In complementary fashion, the storekeeper may sum the cost of supplies and tell the customer the purchase price; in addition the storekeeper takes the payment from the customer and produces change. It is this latter act that interests us here—the player's production of an overpayment and the opponent's production of change. This joint accomplishment of the payment problem constitutes a remove–replace strategy "stretched over" or distributed among the two members of the dyad.

In the following excerpt, we see how thematic roles lead two girls, Veronica and Toni, to form and accomplish distinct mathematical goals as they jointly work toward solving an indirect payment problem.

On her turn, Veronica decided to buy one map and one parrot at a cost of 6 and 5 doubloons, respectively. Toni, assuming the role of storekeeper, performed the addition, saying "the map is six and the parrot is five" and, counting on her fingers, "six, seven, eight, nine, ten, eleven. Okay, eleven doubloons." Veronica had blocks only in denominations of hundreds and tens and, therefore, could not pay with the exact amount of doubloons. Instead, Veronica gave Toni two 10-pieces and asked for change. Toni handed her the change like a cashier, counting up from the cost as she handed Veronica nine 1-pieces, "twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty."

Although it is Veronica's decision to buy a map and a parrot that initiates the joint problem, it is Toni, in her role of storekeeper, who formulates and accomplishes the mathematical goals of adding the cost of the two items and determining the appropriate change. As customer, Veronica also contributed toward accomplishing the purchase problem: once Toni said that the total cost of her purchase would be 11 doubloons, Veronica formulated and accomplished the mathematical goals entailed in making an appropriate payment.

In Toni and Veronica's interaction, we see that numerical goals and the means for successfully accomplishing them are less complex for the player than for the opponent. In these thematically organized forms of assistance, players' goals principally involve a count of doubloons (respecting their denominational structure) in order to produce an appropriate overpayment. In contrast, opponents are engaged with more complex goals, including those entailed in arithmetical problem solving. An opponent (as storekeeper) may be asked to sum together the cost of several supplies and, if handed a 100-piece, be required by virtue of the thematic role to determine and return change. Such a task involves not only the representation of doubloons in quantitative terms, but also an arithmetical transformation (often accomplished by means of a complex counting strategy). Thus, the incidence of thematic roles is both an index of players' involvement in the distributed form of the remove–replace strategy, and an indication of the differential complexity of the numerical goals that individual players were structuring and accomplishing in joint play.

Fig. 4 contains the percentage of players' turns in which thematically organized assistance occurred in dyadic activity for the four groups of players. To analyze these data, we conducted a 2 (Group: mixed grade v. same grade) $\times$ 2 (Player's grade: third v. fourth grade) $\times$ 2 (Session: first v. last) ANOVA on the percentage of turns with thematically organized assistance. The analysis yielded only a significant Grade $\times$ Group interaction ($F(1,33) = 15.49$, $p < 0.0001$).

Duncan post hoc comparisons revealed that in mixed-ability dyads (third and fourth grade children playing one another) interactions were more likely to be thematically organized during the third grader's turn; that is, thematic roles were more common when the third grader was purchasing supplies than when the fourth grader was purchasing supplies ($p < 0.05$). This means that when third graders purchased supplies, the fourth grade opponent often took on the storekeeper's role and, consequently, carried the weight of structuring and accomplishing the more complex arithmetical goals (calculating costs and change); the third grade player, in contrast, structured and accomplished the less complex overpayment goals. Reciprocally, the relatively few thematically organized interactions when fourth graders were the players in mixed grade dyads means that fourth graders tended to accomplish the transaction on their own.* Fourth graders playing other fourth graders (pairings in which both members likely were able to produce means for

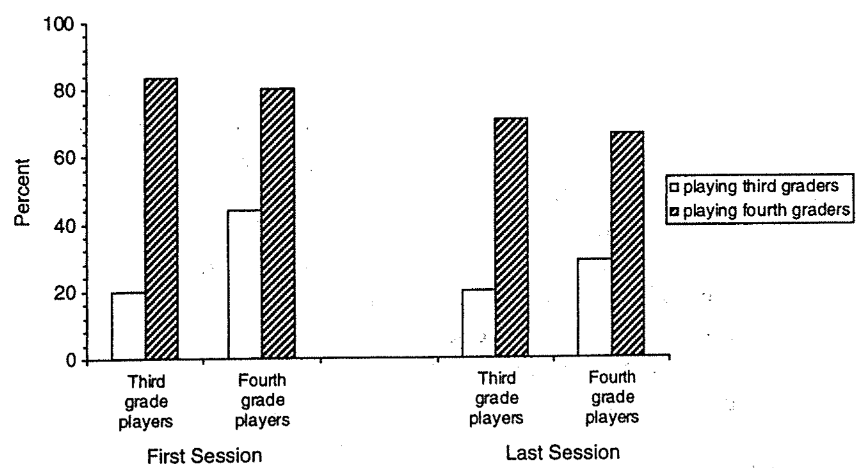

Figure 4. Percent of turns in which dyads used thematic roles in payment problems as a function of players' grade level, opponents' grade level, and session.

*Interestingly, we found that as a collective, third graders playing against fourth graders accomplished correctly more indirect payment problems than did third graders who played against other third graders, apparently because the former, but not the latter, could "off-load" the more complex arithmetical goals onto their fourth grade opponents.

accomplishing the complex arithmetical goals) used thematic roles frequently during their turns.

Children's Solutions to Base-10 Block Arithmetical Problems Following Play

We now turn to how participation in the joint activity of play influences children's learning. For illustration purposes, we limit our discussion to children's solutions to a post-test arithmetic problem with base-10 blocks.

The post-test problem is similar to the indirect payment problems that emerged frequently in play (discussed above), although the subtrahend (228) is greater than the values encountered in Treasure Hunt. In the problem, a child is presented with three 100-pieces, five 10-pieces, four 1-pieces, a "bank" of additional pieces, and is told, "Show me how much you have left when you take away 228. You can use these blocks over here [pointing to the bank] to help you trade to solve the problem if you'd like." Interviews were conducted with the third and fourth grade children who had played Treasure Hunt and with children matched for mathematics achievement from the same classrooms who had not played. To familiarize the non-players with the base-10 block materials, an interviewer taught children the values of the blocks and the equivalence relations between the blocks. Once these children reached a criterion, post-testing began.

We coded three types of solutions to this problem that were similar to the collective solutions to the indirect payment problems that emerged in play: inadequate solutions, adequate solutions that made use of a "trade–pay" strategy, and adequate solutions that made use of a "remove–replace" strategy. (Schematics of the latter two solutions are depicted in Fig. 3.)

Support for a Developmental Sequence Through an Analysis of Non-Players' Strategies

Since the exact payment strategy is the easiest way to accomplish payments, we expected that many children would solve indirect payment problems by first converting them into direct payment problems through trading. As in the trade–pay strategy of Fig. 3, a child would first trade 1 ten-block for 10 one-blocks and, with the resulting distribution of smaller denominations, then produce an exact payment. Only later would children abbreviate the process, embedding the trade in the remove–replace strategy.

Fig. 5 contains the percent distribution of strategies for players and non-players. We use these results to inform our analysis of the development of children's base-10 block arithmetical strategies, and to evaluate the role of children's play in their developing strategic knowledge.

The performances of third and fourth grade *non-players* represented in Fig. 5 provides "baseline" information that shows some support for our argument for a developmental ordering. At third grade, non-players who produce adequate solutions tended to use trade–subtract strategies, whereas at fourth grade, non-players who produced adequate solutions tended to use remove–replace strategies.* Thus, both logical and empirical analyses sup-

*Because of the preponderance of inadequate solutions in the non-players groups, it wasn't possible to test these contrasts.

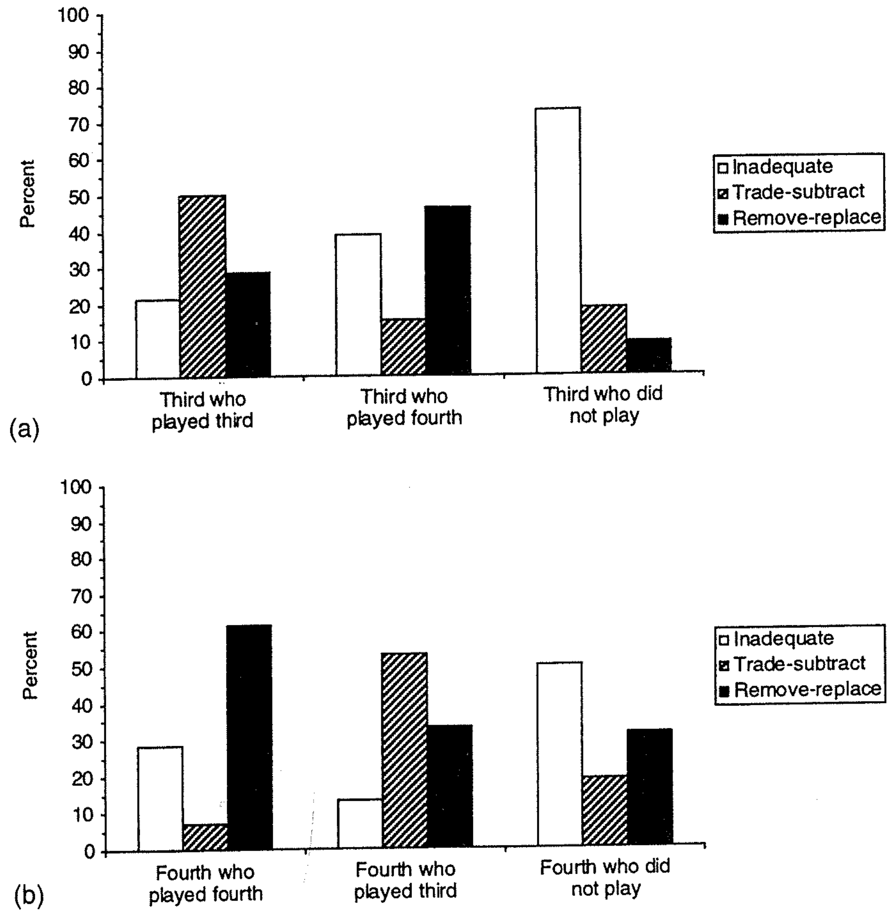

Figure 5. Percent distribution of strategies used by players and non-players on the post-test. (a) Third graders. (b) Fourth graders.

port the claim that the trade–pay strategy is a precursor to the use of remove–replace strategies.

The Influence of Play on Children's Developing Strategic Knowledge

We now turn to our analysis of the role of play in children's strategic knowledge. A comparison of the percent of inadequate strategies for players and non-players in Fig. 5 shows that game play supported children's production of adequate solutions: players as contrasted with non-players produced fewer inadequate solutions at both third grade (Fig. 5a (chi-square (N = 38, d.f. = 1) = 5.95, $p < 0.05$)) and fourth grade (Fig. 5b, (chi-square (N = 45, d.f. = 1) = 4.13, $p < 0.05$)).

In addition to facilitating children's accuracy on the post-test, game play appears to have altered the normative sequence in children's acquisition of strategic knowledge. Here we return to the importance of analyzing emergent goals in practices as a basis for understanding children's learning in collective activity. Following this analytic tack, we expected that goals which emerged in the play of Treasure Hunt as a consequence of thematic roles might alter the expected progression from trade–pay to remove–replace strategies. We sketch these expectations in Table 1.

We reasoned that third graders who played fourth graders had regularly engaged in the distributed remove–replace strategy during play and, therefore, had many opportunities to construct overpayment goals and to make sense of the change they received in relation to both those goals and the cost of their purchase. In contrast, third graders who played against other third graders did not have such opportunities and, instead, had to fall back on exact payment goals, creating equivalence trades (e.g., one 10s block for ten 1s blocks) to accomplish such payments (trade–pay strategies). We suspected that these different patterns of emergent goals in play would lead to group differences in strategy use on the post-test. Fig. 5(a) supports this analysis. Third graders who played fourth graders differed from third graders who played third graders in their distribution of adequate strategies (though Fisher exact test indicated only a marginal level of significance (Fisher ($N = 19$, $p = 0.11$)). For third graders who played fourth graders, the modal strategy was remove–replace; in contrast, for third graders who played third graders, the modal strategy was trade–pay.

We applied the same rationale to understand fourth graders' post-test performances. When fourth graders played third graders in Treasure Hunt, they rarely used thematic roles; consequently they had little opportunity during play to construct the overpayment goals entailed by the distributed remove–replace strategy. Therefore, they, too, might use trade–pay solutions on the post-test as they had done during play. In contrast, fourth graders who played other fourth graders used thematic roles frequently for indirect payment problems, providing them with many opportunities to construct goals linked to the remove–replace strategy in play and to make sense of the change received with reference to these overpayment goals. We expected, therefore, that fourth graders playing fourth graders would then draw on this strategy in the post-test. The contrasts in Fig. 5(b) confirm these expectations. Fourth graders who played third graders differed from fourth graders who played fourth graders in their distribution of adequate strategies (Fisher ($N = 23$, $p = 0.017$)). For fourth graders who played third graders, the modal strategy was trade–pay; in contrast, for fourth graders who played fourth graders, the modal strategy was remove–replace.

Table 1
The Predominant Social Organization of Play and its Relation to Problem Solving Strategies During Play and on the Post-Test for Four Groups of Players

Group	Social organization of play	Solution strategy during play	Individual post-test strategy
Third playing third	Infrequent use of thematic roles	Trade–pay	Trade–pay
Third playing fourth	Frequent use of thematic roles	Distributed remove–replace	Remove–replace
Fourth playing third	Infrequent use of thematic roles	Trade–pay	Trade–pay
Fourth playing fourth	Frequent use of thematic roles	Distributed remove–replace	Remove–replace

Concluding Remarks

In his explication of Activity Theory, A.N. Leontiev (1981) remarked on the emergence of divisions of labor in the history of societies.

> The emergence in activity of goal-directed processes or actions was historically the consequence of the transition of humans to life in society. The activity of the participants of collective labor is induced by its product, which initially met the needs of each participant directly. However, the emergence of even the simplest technical division of labor necessarily leads to isolation of the separate partial results, which are achieved by the separate participants in the collective labor activity, but do not *in and of themselves* satisfy their needs. Their needs are not satisfied by these "intermediate" results, but by the portions of the product of their aggregate activity that each participant receives on the basis of the relations with each other during the labor process, i.e., on the basis of *social* relations. (p. 60; emphasis in the original)

Though geared toward historical issues, Leontiev's remarks bear in an interesting way on two of our principal concerns: exploring the reflexive relation between individuals and the collective in joint activity, and understanding the learning that ensues from this relation.

In our framework, children's solutions to game-linked problems are analogous to Leontiev's "products." Problems—such as the purchase of supplies—anchor individual activity in relation to a collective concern: completing a purchase allows play to move ahead. As children work toward accomplishing problems, individual players' goals and the means of accomplishing them play off one another as players achieve intermediate—or what Leontiev termed "partial"—results toward problem resolution (e.g., identification of what to purchase, construction of a payment, return of change). In their efforts to resolve joint problems in the flow of loosely coupled goal-directed activity, participants may create seamless collective solutions.

Our analyses of Treasure Hunt point to various factors that influence the formation of mathematical goals in collective activities. First, children's prior mathematical understandings appear to have influenced their adoption of thematic roles in play. The third grade players' lower levels of mathematical ability led them to rarely assume the role of "storekeeper," whereas fourth grade players' competence at handling cost and change problems led them to assume such roles frequently. Second, the artifacts used in Treasure Hunt—like the gold doubloons—also influenced the emergence of thematic roles in play. If doubloons had no denominational structure, then indirect payment problems—a problem type that supported a differentiation of goals—would never have occurred in play. Finally, the activity structure of play, in which social norms supported the distribution of problem solutions, also is implicated in goal differentiation; were children solving arithmetical problems in a traditional classroom, collaboration and the distribution of goals in the solution of an arithmetical problem likely would have been discouraged.

The analytic concern for understanding the interplay between collective activity and individual development is central both to the burgeoning field of cultural psychology and fundamental to understanding children's learning in educational practices. In many ways, Treasure Hunt has presented an ideal focus for gaining insight into the character of this interplay. The fact that group size is limited to dyads, that the game is well structured, and that the domain of knowledge—children's arithmetic—is relatively well understood, work to support the focused and productive analyses. Given the payoff with Treasure Hunt, we suspect that an extension of distinctions between collective problems and individual goals perhaps may be profitably applied to more ill-structured practices in which constraints that we enjoyed in the analysis of Treasure Hunt are relaxed.

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