CHAPTER **7**

The Development of Numerical Cognition: Cross-Cultural Perspectives[1]

This chapter is about the relation between culture and the development of numerical thought. It is the only chapter in this volume with a specific focus on culture, and for this reason, we believe that it serves an important function. It is a reminder that the development of numerical thinking is embedded in social life, and the way children form and apply numerical concepts is partly rooted in their social experience.

The chapter is divided into three sections. In the first section, we discuss some of the general properties of numeration systems and the way they are employed to represent number. In the second section, we review two major theoretical formulations of cognitive development, one advanced by Vygotsky, the other by Piaget. Each provides a basis for inquiry into universal and culture-specific processes in children's formation of numerical concepts. In the third section, we examine the strengths and weaknesses of the existing research on cross-cultural number development associated with each theoretical formulation. Finally, in our concluding remarks, we offer some suggestions concerning fruitful new research directions.

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THE DEVELOPMENT OF MATHEMATICAL THINKING

NUMERATION SYSTEMS

Virtually all cultural groups have developed or borrowed numeration systems for the purposes of communicating numerical information and mediating numerical problem solving. Despite the diversity of these systems and the purposes for which they are used, the organization of any numeration system can be understood in terms of a distinction between number symbols and number operations (see Gelman & Gallistel, 1978; Saxe, 1981[a]). This distinction is most easily understood by means of an example.

Consider a young shepherd who is entrusted with the task of comparing the relative size of two flocks of sheep. The shepherd first tries the most direct method of comparison by using a correspondence operation—he tries to establish a one-to-one matching between the flocks. He soon discovers, however, that such a pairing is difficult to accomplish because sheep do not remain stationary. Consequently, he chooses an alternative approach that is less direct but ultimately more powerful. The shepherd solves the task by employing an intermediate group of elements, a set of pebbles. With the pebbles he can represent the correspondence operation symbolically. He first establishes a one-to-one correspondence relation between the set of pebbles and the first flock. He then turns to the second flock. If he can establish an exact one-to-one correspondence relation between the pebbles and the second flock, he can infer that the flocks have the same number; if not, he can determine which flock has the greater number.

This example highlights several properties of the activity of number representation. The first is that individuals can and often do employ aspects of their environment (the pebbles) as symbolic vehicles in order to increase the power of their problem solving. In particular, the use of pebbles as an intermediary enables the shepherd to compare sets that are distant in space (or otherwise difficult to physically pair.) The second property is that individuals employ symbolic vehicles to represent logico-mathematical relations—relations that are not in the objects but are an inherent aspect of the subject's enumerative activities. In other words, a summation of the sheep is not in any way a feature of the sheep themselves, nor is it inherent in the shepherd's vehicle of representation. Rather, it is an abstract transformation imposed by the shepherd on the sheep. The third property illustrated by the example is that the intermediary, which initially has only a local function—to compare the two sets of sheep numerically—can become a symbolic object with which the individual himself interacts. For example, the shepherd can perform additions and subtractions directly on the pebbles in order to signify operations that could be performed on any set of discrete elements, real or imaginary. These numerical relations would be inconceivable without their embodiment in a representational system such as pebbles. Thus, the problem-solving intermediary and the numerical operations can interweave with one another in such a way that they become functionally indissociable.

Although these properties have been presented in the context of a shepherd's idiosyncratic form of number representation (the pebbles), the same properties are evident in an individual's use of culturally defined forms of numeration (such as the Western numeration system). However, in the latter case, the symbolic vehicle is a knowledge system that has evolved in social history to serve the collective needs of members of a particular cultural group.

DEVELOPMENTAL MODELS

Each of the three properties just discussed raises a set of fundamental questions about the development of numerical thought, questions that will provide a direction for the discussion to follow. Property 1 implies that during the course of development, children transform historical forms of number representation that are initially external to their cognitive repetoire into symbolic vehicles that become an inherent part of their problem-solving activities. This means that the study of number development must not only describe the child's acquisition of the numeration system of the culture, but more importantly, it must provide an analysis of the changing relations over the course of the child's development between the child's acquisition of the numeration system and the process of problem solving. Property 2 implies that children use number symbols to represent logico-mathematical operations such as addition, subtraction, multiplication, and division. Thus, the study of number development must also provide an account of the origins and development of these logical operations and the way they become manifest in the use and understanding of a numeration system. Lastly, Property 3 implies that numerical thought often entails operating with and upon a particular historical invention, the form of which may facilitate operations of certain types and limit others. For this reason, the study of number development should provide an account of the way that differences in the sociohistorical constructions of number will lead to variations in the ways individuals solve problems.

In this section, we will consider two theoretical formulations of cognitive development, one inspired by Vygotsky, the other by Piaget. Each, we believe, offer important insights about number development that address, with varying degrees of adequacy, these three issues.

Vygotsky's Approach

The Soviet approach, represented in the writings of L. S. Vygotsky (1962, 1978), is concerned with understanding cognition as a "mediated" activity. By "mediated" it is meant that individuals do not interact with the world directly,

instead they interact with their personal representations of the world. These representations include such social constructions as linguistic signs and discourse, orthographies, and numeration systems. For Vygotsky, the formative processes that influence the development of representational activities and the organizing role that these activities have for the general development of intelligence provide a critical area for study. It is these formative processes that link the cognitive development of the individual with the collective practices of the social group. Though Vygotsky did not treat the development of numerical cognition directly in his work, he did offer a general treatment of language and its relation to thought. A brief outline of his developmental framework for language will provide an introduction to our extension of Vygotsky's approach to the development of number.

Vygotsky argued that an inherent property of the development of speech and thought is that the relations between them change. Early in development, speech and thought are rooted in different kinds of activities and develop independently of one another. Preverbal thought consists of practical goal directed action. At the same time, early vocalizations are prerational and serve a primarily expressive function. With development, the child gradually transforms the historical construction of language into a vehicle that mediates his or her own thinking, and at the same time, the child's thinking becomes a mediated activity linked to the social group. Vygotsky is careful to point out that the interpenetration of thought and speech is never complete; rather, there is a dialectical relation between the two throughout development such that the emergent properties of their union are continually changing.

If we extend Vygotsky's treatment to numerical cognition, his approach leads to an analysis of two of the three core issues presented in the example with the shepherd—in particular, how it is that a representational system for number, which has emerged in the social history of a cultural group, is transformed by the individual such that it becomes an intermediary deployed in problem-solving activities (Property 1) and a symbolic object with which an individual interacts (Property 3).

NUMERATION SYSTEMS AS PRODUCTS OF SOCIAL HISTORY

Since, in Vygotsky's formulation the acquisition of historical systems of representation should profoundly influence the cognitive development of the individual, it is important to consider the differences in the characteristics of numeration systems across cultural groups as well as some of the factors that contribute to the changes in numeration systems in the course of the social history of a group.

For our purposes, one way of imposing some order on the wide range of numeration systems is to categorize them on the basis of two main features—the

vehicles used to signify numerical relations (physical or verbal representations) and the predominant organizational structure of the system (base or nonbase structure). We will briefly review examples of each of the four resulting categories.

Spatial Representation/No Base Structure.   Typical of numeration systems that employ physical entities for number terms and have no base structure are the body part counting systems used by various New Guinea highland groups (see Lancy, 1978). The Oksapmin of the West Sepik Province employ such a system (see Saxe, 1981b). Twenty-seven body parts are used to represent number, 13 on each side of the upper periphery of the body, and the nose. To count, the Oksapmin name each body part in a prescribed order beginning with the thumb on one hand and ending with the little finger on the other. Thus, *besa*, the word for forearm, denotes the number "7" when counted on one side of the body and the number "21" when counted on the other side. If an individual needs to enumerate beyond "27," the count continues back up the second side of the body. This system does not have a base structure, and it provides only a finite set of terms by which to represent numerical relations. Numeration systems organized in this way are rather uncommon today; Lancy (1978) estimates them to represent only 10% of the New Guinean counting systems.

Spatial Representation/Base Structure.   The second category of numeration system also employs physical–spatial terms to represent numerical relations, but differs from the first in that a base or generative structure is employed.

An excellent example of how changing economic conditions may lead to an organizational change in the form of counting systems comes again from the Oksapmin, who in recent years, with the introduction of Western currency to the area, have adapted their indigenous system to include a base structure. This adaptation has enabled individuals to represent much larger numbers, an essential requirement for accurately carrying out economic transactions with currency. In the adapted system, the count begins as it does traditionally, with the thumb; however, instead of proceeding to the little finger on the other side of the body (27), the individual stops at the inner elbow of the other side (20). The completion of the count at this point corresponds to the number of shillings in a pound. When there is a need to count further, one round or "one pound" is recorded and the count is begun again. The system, then, is a hybrid that retains its signifiers for numerical relations but makes use of the base structure inherent in the local system of currency (see Moylan, forthcoming; or Saxe, 1982, in press, for a more extensive discussion of this system).

Nonspatial Representation/No Base Structure.   Most examples that fall into this category are not true numeration systems; that is, they are not employed for representing cardinal number, although they do convey ordinal relations. The days of the week are a typical example. The days are ordered as a list of primary lexical terms and are used to designate the ordinal relation between cyclical 24-

hour periods. No base is involved, although the terms are used iteratively.

Similar ordinal systems are used for other relational purposes. Saxe (1981c) reported that the Ponam islanders name their children by using a system that marks birth order relations. For example, in all families, boys are given one of a set of eight names, depending upon the order of their birth with respect to other boys in the family; there is a different set of eight names that apply to girls. Like the days of the week, the birth order naming system does not encode cardinal relations but is used to designate order relations.

Nonspatial Representation/Base Structure. Verbal numeration systems, such as those employed in Western societies, use arbitrary number names to signify the units and employ a base principle to generate higher numbers. Unlike written place value numerals, which are rare inventions, verbal numeration systems of this type have been developed in many cultures and were in existence long before written numeration systems of a similar structure were invented.

The most common numeration system of this kind employs a base ten or twenty, often with vestiges of a subordinated base five. Zaslavsky (1973) reports that these two types predominate in Africa. The Dioula of the Ivory Coast have a prototypical base ten system in which the number words express the additive function directly: "Tan nin kele" (10 + 1), "Tan nin fla" (10 + 2), and so on (Posner, 1982). The Dioula rely on the regular structure of their numeration system in carrying out mental arithmetic. Adding and multiplying is done by regrouping numbers by tens. Thus, 56 + 49 would typically be added in the following manner: (50 + 40 = 90; 6 − 1 = 5; 1 + 9 = 10; 90 + 10 + 5 = 105). The formation of numerals in this exceedingly regular base ten system is suggestive of such a procedure.

An unusually complex base twenty system that employs both additive, multiplicative, and subtractive principles is that used by another West African group, the Yoruba of Nigeria. Consider the Yoruba verbalization of the number "315." It can be expressed as follows: (200 × 2) − (20 × 4) − 5. The expression for 525 is (200 × 3) − (20 × 4) + 5 (Armstrong, 1962; Zaslavsky, 1973). This system is not one reserved for elites, though it would seem to require considerable arithmetic dexterity to master. The Yoruba are urban people and accustomed to commercial interactions. Learning to count and to use the Yoruba number system for arithmetical problem solving apparently occurs in the context of everyday market activities.

Although base ten and twenty constructs dominate the field of world numeration systems (Smeltzer, 1958), a variety of unusual bases have been documented both in contemporary and ancient groups. A small sample of these include a base four system used by the Huku of Uganda (Zaslavsky, 1973), a base fifteen system used by the New Guinea Huli (Cheetam, 1978), and a base sixty system used by the ancient Babylonians (Menninger, 1969).

Our brief survey of different types of numeration systems highlights a major

theme of Soviet psychology, namely, that children's cognitive development occurs within a social context that itself has evolved varying intellectual tools. The way that these tools become the child's own and mediate numerical problem solving are central issues for Vygotsky's formulation. Indeed, Vygotsky's analytic categories are geared toward illuminating this fundamental topic in cognitive development. We consider Vygotsky's formulation next.

THE ACQUISITION OF KNOWLEDGE SYSTEMS

A major theme of Vygotsky's developmental approach is that there are qualitative changes in children's use of culturally organized knowledge systems for problem-solving activities and that these changes are an inherent feature of the acquisition process itself. To explain this process from Vygotsky's perspective, it is important to distinguish between two types of learning experiences, those that occur from the "bottom up," giving rise to what Vygotsky has called *spontaneous concepts,* and those that occur from the "top down," producing what Vygotsky has called *scientific concepts.*

Bottom-up learning is described as resulting from the child's spontaneous attempt to understand aspects of social and physical reality without the direct aid of adult or peer tutoring. These types of experiences lead the child to acquire practical concepts; that is, the child achieves local solutions to particular problems. For example, in order to reproduce the number of beads in a set, a child might create a numerically equivalent set by establishing one-to-one correspondences. In contrast, top-down learning is described as resulting from interactions with adults or more capable peers. In these interactions, problems are posed for the child, and he or she is presented with concepts of general applicability that are valued in the culture. Top-down learning, such as that encountered in school, gives the child the opportunity to form general concepts that may be adapted to different problem types but are not necessarily grounded in immediate experience. For example, a child might learn how to use the Western counting system in order to solve numerical problems. According to Vygotsky's formulation, each kind of learning should inform the other, and these links can be an important mechanism of developmental change.

Recent efforts consistent with Vygotsky's approach have attempted to demonstrate that a fundamental aspect of adult–child interactions in problem-solving contexts consists of a "scaffolding" process (Wood, Bruner, & Ross, 1976), in which adults adjust their (top-down) dialogue in such a way that the child can relate his or her (bottom-up) experiences to the novel problem at hand (Gearhart & Newman, 1980; Wertsch, 1979; Wood et al., 1976; Wood & Middleton, 1975). Wertsch, for instance, has demonstrated four levels of scaffolding behavior apparent in mothers' interactions with their toddlers who are attempting to solve a puzzle. This scheme can be applied to the child's acquisition of numera-

tion concepts in the following way. Early in the developmental process, adults engage children in number-related activities that children are not completely capable of doing on their own, but are within the grasp of their understanding—activities that are within what Vygotsky labeled *the zone of proximal development*. By guiding them through number tasks, adults introduce children to the number symbols and general numeration strategies that are specific to their cultural group. Although at first the strategies and symbols are regulated by the adult, over the course of development the child is able to integrate and connect these concepts with those derived from his or her own (bottom-up) experiences and thereby achieves a progressive understanding of the form and function of these symbols and strategies. With increasing experience in the form of interaction with adults and others, top-down concepts come to guide and organize bottom-up learning with the result that children's problem-solving behavior becomes increasingly independent of others and yet more closely mirrors the cognitive functioning of members of their cultural group.

This Vygotskiian-inspired account offers some important insights about the function and influence that mediational systems have on the child's developing number abilities. First, central to Vygotsky's account is a materialism. The subject organizes the material available in the environment in order to increase the power of problem solving. Some of this "material" is social and takes the form of culturally organized forms of representational systems. Thus, just as our shepherd transformed pebbles into a mediational system, the child transforms socially defined symbol systems for number into a vehicle for numerical representation (Property 1). Second, on the Soviet account, the nature of this material and the way it is used in cultural life in turn influences the character of numerical thought in a particular way (Property 3). Thus, the sociohistorical context of the child's development plays a powerful determining role in the character of the numerical thought of the child. Although the Soviet approach presents a treatment of the first and third properties of numeration, it does not offer an account of the second property—the logical foundations of numerical representation and numeration concepts. It is the Piagetian formulation that addresses these issues primarily, and by so doing, deemphasizes the role of specific cultural forms of representation in structuring the child's numerical concepts.

Piaget's Cognitive Developmental Theory

Piaget's theory is concerned with the origins of logical structures of thought and the characteristics of these structures. The major analytic categories of the theory as well as the empirical work that the theory has generated are focused on this core issue.

Central to Piaget's thinking is a rejection of both nativistic and empiricist formulations of the origins of logical structures. Instead, he argues that the origins of logical structures are elaborated in sensorimotor activities, which, through an epigenetic process, are transformed into mental operations in the course of development (see Langer, 1980). During the sensorimotor stage (infancy), these coordinations are practical and directed toward achieving immediate goals. As development proceeds, the logic of practical action is transformed into a logic of reversible concrete operations (during middle childhood). This stage is characterized by a set of classificatory and relational structures (formalized as logico-mathematical groupings) which, Piaget argues, are the basis of children's understanding of elementary numerical operations such as addition, subtraction, multiplication, and division. It is not until the stage of formal operations (adolescence and adulthood), however, that these two sets of classificatory and relational structures become integrated into a generalized reversible system. Piaget argues that this integration is the defining characteristic of hypothetico-deductive reasoning, the essence of mature cognitive functioning. It is Piaget's position that the progression from one stage to another is not the simple result of maturational, experiential, or social factors (such as the acquisition of a numeration or other symbol systems). Rather, progress is achieved by the child through an equilibration process whereby all the aforementioned factors exert an influence as the child strives to achieve a coherence between the dialectic of existing forms of understanding and new experience. Piaget's general definition of the problem of cognitive development, then, differs from Vygotsky's. Whereas Vygotsky's focus is on numeration as a mediated activity (Properties 1 and 3) that has its roots in social interaction, Piaget's focus is on the emergence of logico-mathematical structures that underlie the use of numeration (Property 2) and have their roots in sensorimotor activities.

As a part of his theory, Piaget has offered a developmental analysis of such logico-mathematical concepts as cardinal and ordinal number, the composition of numerical relations, and a wide range of measurement concepts (see Piaget, 1952; 1968). Piaget argues that developmental changes in each of these concepts is a manifestation of a general shift in the organization of logico-mathematical thought. We will review here, in some detail, Piaget's treatment of numerical equivalence (the conservation of one-to-one correspondence relations) that develops with the concrete operational stage, as it is this concept that has been examined most extensively in cross-cultural research.

To investigate developmental changes in children's understanding of the conservation of numerical equivalence, Piaget presented children with a model set of elements and asked them to construct a numerical copy with an available set. Piaget found that, during an early stage, children make judgments of equivalence on the basis of one-way functions, operations which have no inverse. On

this basis, the child carries out a systematic evaluation of the quantities according to the spatial extent of one collection relative to the other ($x \rightarrow y$). For instance, a child might align the endpoints of two sets and disregard the discrete number of the elements in the two sets. Similarly, a child might reason that one collection contains more because it extends further than the other. At the next stage, children begin to coordinate evaluations based on two one-way functions. Thus, children begin to coordinate a numerical evaluation based on length, with one based on density. As a result, children begin to produce one-to-one correspondence relations between two sets to determine numerical equivalence. Nevertheless, at this stage, if one row is subsequently spread apart or crowded together, children generally assume that the equivalence relation is not lasting. Finally, as classificatory and relational operations assume their concrete operational form, the coordination of one-way functions is completed. At this stage, children understand the necessity of conservation.

It is important to point out that the operations entailed in the concept of conservation are also formally entailed in other fundamental numerical concepts such as associativity, commutativity, and transitivity. Thus, whether one spreads elements apart as in the classic conservation task, regroups them (assocativity), rearranges their order (commutativity), or uses an intermediate group of elements as a basis to compare two sets, at a formal level of analysis one is asking a subject about the same or related sets of operational structures. From the Piagetian perspective, understanding any one of these operations by definition implies understanding the others. When subjects exhibit differences in their ability to demonstrate an understanding of related concepts on assessment tasks, variation in performance is ascribed to differences in the way the object "resists" the child's understandings. That is, it is argued that with some materials and in some contexts, it is easier for a child to apply conceptual achievements than in others.

In his approach to cognitive development, Piaget subordinated experiential or cultural factors to the equilibration process. This has probably contributed to the lack of concern with treating experience, in particular cultural experience, as a differentiated theoretical construct. Genevans generally acknowledge the influence of culture as a factor that may influence the rate of progress through the stages or the achievement of the endpoint of the developmental process (i.e., formal operations). Yet they do not explicate the manner in which cultural factors contribute to the developmental process. Insofar as numeration systems are bodies of knowledge that are acquired from one's social group, they, like other collective practices, are understood to have relatively little influence on the way individuals construct numerical concepts. Thus, while offering a formulation about the way in which numerical operations develop, the Piagetian focus leaves unanalyzed the mechanisms by which social factors contribute to the formation of numerical thought.

CROSS-CULTURAL RESEARCH ON THE DEVELOPMENT OF MATHEMATICAL CONCEPTS IN CHILDREN

The Genevan and Soviet formulations each imply that there are universal and culture-specific processes in cognitive development, although the nature of these processes differ in each theoretical formulation. Cross-cultural research has often been informed by one or the other of these theoretical positions, either directly or indirectly, and research on numerical cognition is not an exception. We have organized our review of this literature accordingly and will consider a sample of studies motivated by Piaget's theory as well as selected studies cast in a Vygotskiian theoretical framework. In what follows, we attempt to characterize the researcher's view of the way in which culture interacts with the development of quantitative thought and how the research contributes to each theoretical formulation.

Empirical Work Consistent with the Cultural Practice Approach

The theory proposed by Vygotsky and his student Luria has been subsequently elaborated by a group of American psychologists (Cole, Gay, Glick, & Sharp, 1971; Cole & Scribner, 1974; Wertsch, 1979). It is important to point out that Cole's group has focused on certain of Vygotsky's theoretical constructs and not others. Cole's emphasis has been primarily on a contextual analysis—an analysis of the way the conceptual abilities of individuals are influenced by and adapted to particular problem-solving contexts (a version of Property 3), whereas Vygotsky, in addition, stressed a developmental analysis of the shifting functional relations between different aspects of problem solving over the course of development (Property 1). For the most part, the existing cross-cultural research follows the contextualist focus rather than the developmental one.

Two general types of methodological approaches can be distinguished among those who use a contextualist focus, both of which concern the influence of cultural variables such as schooling, economic specialization, and the form of numeration system on problem solving. However, the first type generally makes use of "culture" as an explanatory construct for differences in the way that individuals mediate their solution to numerical problems, whereas the second makes use of a more fine-grained analysis of the strengths and limits of mediational strategies that develop in particular cultural contexts.

Consider the first type of methodological approach. The logic of these studies follows from the position that societies develop different technologies or

systems of knowledge in response to environmental factors. These bodies of knowledge accrue over the course of history and are transmitted to young members both explicitly (by means of specific childrearing practices) and implicitly (in the way the society organizes the activities that children experience). It is therefore hypothesized that groups with varying cultural practices should solve mathematical tasks differently from one another (see Laboratory of Comparative Human Cognition, 1979, for a comprehensive discussion).

A pioneering effort in the area of cross-cultural numerical cognition that assumes this approach was carried out by Gay and Cole (1967). They examined number and measurement concepts of the Kpelle, a group in Central Liberia, as well as their childrearing practices and traditional educational values in an attempt to determine if these were incompatible with formal arithmetic concepts taught in Western-style schools. They also conducted a series of psychological experiments that examined the Kpelle's skill at estimating quantities of different types. In general, they found that the requirements of everyday life dictated which numerical estimation skills were most fully developed. For instance, the Kpelle were better than American college students at estimating the number of stones in a pile, whereas American college students were better at measuring length by hand spans. Gay and Cole argue that these patterns of differences resulted from the fact that stones are commonly used by Kpelle for tallying the number of cups in a container of rice, as rice is an important staple for the Kpelle. Thus, Gay and Cole argue, the Kpelle excelled on the estimation tasks. In contrast, measuring length by hand spans and foot lengths, a system also in traditional use, was more difficult for the Kpelle than for the Americans. The reason for this, Gay and Cole claim, is that the Kpelle do not have a system for length that employs interchangeable units (e.g., 1 ft = 12 in.). The Americans, on the other hand, estimated the length more accurately, because they used the English system to translate hand spans into inches.

A series of studies conducted on the Ivory Coast also begins with the assumption that significant societal values and cognitive skills acquired by children in a culture are derived in part from the culturally organized activities and practices. The research systematically documents the development of addition problem solving and other mathematical skills, which occur both informally (Ginsburg, Posner, & Russell, 1981a; Posner, 1982) and as a result of schooling (Ginsburg, Posner, & Russell, 1981b). Subjects came from groups with different economies: merchants (Dioula) and agriculturalists (Baoule). It was hypothesized that unschooled children from the merchant culture would be more accurate and use more efficient strategies than the children from agricultural families, because mercantile society values mathematical skill and affords experiences that encourage its development. As expected, the results show that unschooled Dioula (merchant) children adopt more economical strategies than their unschooled Baoule (agricultural) counterparts. In particular, they use a greater number of

memorized addition facts and regrouping by tens ($7 + 5 = 10 + 2$), as compared to the Baoule. Schooling provides requisite experience with number for the Baoule who perform in most respects on a par with Dioula peers when they receive instruction.

The experiment that examined the assimilation of school mathematics by Dioula and Baoule children also provides a result that links the social context of development to the formation of numerical problem-solving strategies. When doing written arithmetic, children initially adopted informal counting methods and, in some cases, traditional regrouping procedures, but with increased school experience, they used the standard written algorithm almost exclusively. Together, these two sets of studies with Ivory Coast populations demonstrate how cognitive strategies for arithmetic problem solving are acquired both within and outside the school context. More generally, the research shows how individuals develop the symbolic skills that are most useful to them in their differing social contexts.

The difference between the studies just described and those comprising the second type is primarily methodological yet important to highlight, as it reflects a shift in the way in which cross cultural research questions are posed (see Glick, 1981, for a recent discussion). In the previous studies, "culture" was offered as the "explanation" of variable performance across population groups. In the studies we review next, the focus is on the experimental situation itself, and group comparisons, if employed at all, occur between closely related populations that differ on a well delimited dimension. A basic theme in these studies is that it is incumbent on the researcher to design an experiment that will draw out the cultural knowledge of subjects. As the experimental task is varied, the strengths, limitations, and characteristics of subjects' cognitive skills are progressively revealed. In this way, it becomes possible to evaluate more effectively the types of mediational strategies and problem-solving techniques specific to individuals in particular cultural contexts. Research of this type, carried out with several of the same ethnic groups as those in the Cole and Ginsburg studies, is described next in some detail.

Lave (1977) sought to determine whether problem-solving strategies learned in school would, as has frequently been suggested in the literature (Greenfield & Bruner, 1969; Scribner & Cole, 1973), transfer to unfamiliar problems that arise in informal learning environments, such as the tailor shop. In other words, Lave was testing the claim that schooling promotes generalized cognitive skills. The study was conducted with a group of tailors from the Vai and Gola tribes whose tailoring experience and years of formal schooling varied independently. Lave's tasks included arithmetic problems that resembled those practiced at school, those typically encountered by tailors in their work, and others which lay somewhere in between the two domains. She argued that her findings support what Cole and his colleagues call the cultural practice or func-

tional learning approach to cognition. Specifically, Lave found that the extent of school experience was the best predictor of success for school-like problems, whereas successful resolution of tailoring problems was predicted best by years of tailoring. Thus, the findings point to the compartmentalization of skills or mediational strategies in particular contexts, an important finding and one that is at odds with previous interpretations of the influence of schooling on cognitive abilities (see Laboratory of Comparative Human Cognition, 1979).

Using both a descriptive and experimental approach, Lave in collaboration with Reed (Reed & Lave, n.d.), explored critical linguistic and experiential contributions to the Vai and Gola tailors' arithmetic knowledge. Based on a prior analysis of the numeration systems used by the tailors and the kinds of arithmetic problems they commonly encountered in different contexts, Reed and Lave were able to predict the type of errors committed by tailors with varying amounts of schooling. For example, unschooled tailors who use the Vai–Gola numeration system, which employs 5, 10, and 20 as a generative base (e.g., $75 = [3 \times 20] + [10 + 5]$, often adopt a regrouping strategy for addition that creates groupings around each of these relevant numbers (e.g., $17 + 3 = [10 + 5 + 2] + 3 = 10 + 5 + 5 = 10 + 10 = 20$). The propensity for "losing track" of the groups to be summed when complex numbers are involved is very great. By contrast, individuals with 5 or more years of schooling adopt school-like algorithms (e.g., those for column addition) and typically make "carrying" errors that are off by factors of a greater magnitude than their unschooled colleagues (e.g., a factor of 100 in a three-digit problem). Evidence that tailors with just a little schooling were switching to the school-like system was also apparent. Thus, the general finding is that schooling has an impact on the way that arithmetic gets done, although it does not necessarily lead to a more generalized understanding of arithmetic principles. Such generalized understanding, the authors contend, is more a function of practice than educational background, though, like other researchers, they do not detail the type of practice required.

Petitto's (1979) thesis work among the Dioula provides some data relevant to understanding the relation between the structure of practical activities and generalized arithmetic knowledge. She studied arithmetic problem solving in two professional groups, tailors and cloth merchants. Both groups typically manipulate cloth, money, and meter measures in their work, but in different ways. For example, the main arithmetic calculations of cloth merchants involve determining the price of fractions of a meter of cloth. This is a ratio problem which requires complex arithmetic. Tailors' use of the meter is more concrete and direct, mainly requiring a simple doubling or halving operation. Both the merchants and tailors were interviewed and administered simple tasks that supposedly tap the component skills required to solve proportion tasks. In addition, subjects were given problems that required using a proportion to determine the price of various lengths of cloth (familiar materials) and comparable quantities of

oranges (unfamiliar materials). There were no differences observed in the component skills tasks between tailors and merchants. However, the cloth merchants outperformed the tailors in both types of proportion tasks, not only on the cloth problems familiar to the merchants. Thus, Petitto's data show that under certain conditions, skills do transfer from familiar to novel contexts.

Another dimension of Petitto's work (Petitto & Ginsburg, in press) concerns the strategies the Dioulas use to perform the four basic arithmetic operations and whether their calculational ability is tied to a tacit understanding of formal mathematical principles. To accomplish this, she devised eight pairs of problems such that for any given pair the solution of the second could bypass calculation by relying on mathematical principles of different types. For example, the law of commutativity could be invoked tacitly to provide an immediate answer to the problem $38 + 46$, after a subject had already calculated $46 + 38$. A similar relationship holds between the problems 6×100 and 100×6.

The results showed considerable variability in the understanding and use of formal principles. For instance, many subjects identified the commutative relationship in addition problems, but failed to do so with multiplication. This failure is attributed to the asymmetry in the Dioula's linguistic expression for multiplication (the gloss for 100×6 is given as 6 "added to itself" 100 times which, if one actually proceeds to add iteratively, is much more laborious than 100 added to itself 6 times). Given the linguistic construction for the terms of the system, it is not obvious to the Dioula that at a deeper conceptual level the multiplier and multiplicand can be exchanged without affecting the product.

The results with respect to calculational strategies corroborate and extend those of Ginsburg et al. (1981b). Petitto finds that most Dioula use a regrouping strategy for addition and that this basic approach is elaborated upon in carrying out the other three operations. Subtraction is conceptualized as addition in reverse, multiplication as repeated addition; and division, the most laborious, is carried out by estimating a number that when multiplied by the divisor yields the original quantity. The answer is then checked by addition and adjusted if inaccurate.

As we have seen, both Petitto and Lave have employed a methodogical approach that differs from the standard comparison of groups across cultures. All experimental contrasts involved comparisons within the cultural groups themselves. Both researchers conducted a series of experiments, and Lave relied heavily on ethnographic analysis in framing experimental questions and designing tasks. There is a great deal of convergence in the two sets of results, which is not suprising, given the similarity of cultural setting (the urban tailor's shop), language (both are of the Mande family), and tasks. Though the question of how mathematical skills are acquired is ignored by the authors, in the case of the Dioula, prior work of Posner (in press) offers the explanation that strategies are acquired by means of direct experience manipulating (e.g., grouping) and count-

ing objects. More observational investigations of the activities that account for children's adoption of particular strategies need to be done, however, if we are to have more than a suggestion of how problem-solving strategies develop.

In general, the research consistent with Vygotsky's approach has provided documentation concerning the way numerical skills are interwoven with particular numerational systems and culturally organized practices. This body of research, however, has provided little information about the process of development of these cultural-specific skills, a central concern of Vygotsky's original formulation. As we turn to the Piagetian inspired research, we will see somewhat the reverse trend in the way in which the theoretical approach is translated into empirical research. Here the major focus is on documenting the characteristics of qualitative changes in cognitive functioning that occur in the course of development and a relative lack of concern with the particular cultural context of development.

Piagetian Studies

The cross-cultural research on number development that is inspired by Piaget's theory can be categorized into three general types. First and most extensive is the body of research concerned with determining the validity of Piaget's claim that his proposed stage sequence for number development constitutes a universal developmental process. Second is the limited research that has been conducted to determine the cultural factors that contribute to different rates of progress through Piaget's stage sequence. Third are the rare studies concerned with determining whether and in what way Piaget's general stages of cognitive development are related to the acquisition of different forms of non-Western numeration systems.

A number of authors have provided reviews that cover the first general theme of cross-cultural Piagetian studies (e.g., Ashton, 1975; Carlson, 1976; Dasen, 1972, 1977a), and the consensus among them is that on the whole, in the range of cultural groups studied, there is support for the universality of development in children's understanding of concepts of number conservation through the concrete operational stage. However, time lags are widely cited, and there is considerable variability across cultures with respect to performance on various tasks within a stage. Sometimes the relative difficulty of tasks within a stage does not correspond to Piaget's findings with Western children.

A study by Etuk (1967) concerning the development of number concepts is a good example of this first class of studies. Etuk interviewed Yoruba schoolchildren from traditional and modern homes, and, guided by the Piagetian formulation, she assessed subjects' abilities to classify, seriate, and conserve num-

ber. She found that the rate of acquiring these concepts was somewhat slower among the Yoruba, particularly among those who come from traditional homes than among Western children, although Yorubas exhibited the same basic levels of understanding. Of more significance to the theory perhaps is the relative difficulty of the three tasks. Although conservation and seriation concepts were generally found to emerge at the same time across the sample, class inclusion was not well understood by any group. In a more recent study, Opper (1977) found a similar pattern for the acquisition of the same three concepts in rural Thai children.

As is the case with much of the Piagetian research conducted during this period, Etuk's study leaves us with very little information concerning the reason for the variation in performance between ''traditional'' and ''modern'' children. This is unfortunate since there are many aspects of Yoruba culture that make it particularly interesting from the perspective of number development. Etuk mentions in passing that the vast majority of the traditional mothers (87%) were petty traders, but she gives no indication of how this kind of environment might be expected to stimulate mathematical activities or why, given the potential advantage, the traditional child performs more poorly than children from more Western families. Moreover, no mention is made of the complicated numeration system used by the Yoruba described in a previous section of this chapter. This oversight may reflect the small role assigned to numeration systems in Piaget's formulation as well as the overriding concern of Etuk's study—documentation of the universal aspects of Piaget's stages, stages that are not expected to be a product of cultural factors.

A second general approach to cross-cultural Piagetian research was adopted as a response to the many studies like Etuk's, which find different rates of development but which offer little explanation, other than the well-packaged independent variable—culture—to account for them. This latter research is concerned with identifying factors that influence the rate of acquisition of logico-mathematical concepts. The notion is that it may be possible to specify the kinds of organism–environment interactions that influence cognitive development. Researchers who have taken this tack have attempted to identify environments that might encourage or discourage the development of specific concepts.

This general line of research was initiated by Price-Williams, Gordon, and Ramirez (1967), who studied the children of traditional potters in Mexico and found them to be more advanced in their ability to conserve quantities of clay than a control group of Mexican children. Other conservation concepts were not affected. Using this study as a model, subsequent research has delimited further the nature of experiences required to bring about cognitive structural change. Steinberg and Dunn (1976), who found no advantage for another group of potters' children in Mexico, point out that the actual experience that these partic-

ular children have with clay does not in fact leave weight or amount invariant. The loss of moisture when pots are fired in the traditional manner causes them to shrink in size and become lighter.

Another conservation study that included potters and their children also examined subjects from commercial and agricultural families. Adjei (1977) expected his Ghanaian subjects to excel on the conservation task most related to their occupational specialties. Although no specific hypotheses were given, the reader assumes that Adjei proposes merchants to be advanced on number tasks and potters on the weight and substance tasks. It is not obvious on which tasks the farming group would be expected to show superior performance. Adjei's results are not in perfect correlation with these predictions. Among the 7- to 9-year-olds, the only difference observed was for weight, on which the potters' children out-performed the other groups. Women potters excelled on the substance, weight, and volume tasks, all skills called upon in pottery making. The number task, however, did not differentiate among women with different occupations; all the groups demonstrated an understanding of number conservation. Although this finding does not support Adjei's initial prediction, it is consistent with the classical Piagetian formulation that number conservation is expected to be acquired before the other conservations. It may be, as Adjei indicates, that in the number conservation task there is a ceiling effect that masks potential group differences. If this were the case, one would expect to see some variation at an earlier age. Although Adjei administered an inequality and seriation task to a group of 4- to 5-year-olds, he unfortunately does not report the results. Thus, whether there is an effect of cultural orientation such as commerce on the development of numerical concepts in young Ghanaian children is left unanswered by this study.

Another set of studies conducted in Papua New Guinea investigated factors that enhance the acquisition of logical concepts. The Papua New Guinea Indigenous Mathematics Project (Lancy, 1978) was motivated by national concern about the poor acquisition of mathematical skills by schoolchildren. Several possibilities were advanced concerning the causes of poor performance. One was that native numeration systems might actually be subverting the acquisition of modern mathematics. The specific hypothesis put forth was that children from societies with "true numerical counting systems" (i.e., composed of a base structure and nonspatial representation), considered by Lancy to be more abstract than other systems in use in Papua New Guinea, would evidence more abstract thinking in general. The nature of the causal relationship between "abstract" thinking and the type of numeration system available in a culture was not specified. It is not clear, for instance, whether the use of a particular type of numeration system is taken to influence "abstract" thinking or whether groups using "advanced" systems are more likely to engage in other activities (like commerce) that may facilitate abstract thought.

In any case, to test the hypothesis, 11 cultural groups with numeration systems of various types were examined; they ranged from those similar to Western systems to others that use body parts. Both conservation and other cognitive tasks were given to schooled and unschooled children from the chosen groups. No clear relationship was found between the type of numeration system in use and performance on any of the general cognitive tasks administered. Although Lancy could not produce evidence supporting his hypothesis, the organization of numeration systems may in fact have more local effects on quantitative thought, as we have seen in Lave's and Petitto's studies concerning adult problem solving.

In another study reported by Posner and Baroody (1979), the economic orientation of one's cultural group was hypothesized to be a factor that influenced the rate of acquiring number concepts. Posner and Baroody (1979) used a design similar to Adjei's in order to investigate the relation between the acquisition of number conservation and the development of counting skill among children on the Ivory Coast. Preschool, schooled, and unschooled children from tribes with different economic orientations (merchant and agricultural) were selected for study. The results showed an interaction between schooling and cultural background. Unschooled merchant children (Bioula) performed on a par with school children who had received explicit instruction in counting and conservation. The unschooled children from the agricultural milieu (Baoule) developed these skills at a slower rate, although those attending school were not shown to be at a disadvantage. Thus, it appears, on the one hand, the activities and requirements of merchant culture play an important role in the development of numerical concepts, and on the other, that instruction provides experiences that are unavailable to children from less quantitative-oriented societies. Even when there is no direct instructional intervention, the activities and requirements of merchant culture can have an effect on the development of numerical concepts.

The role of counting in facilitating the development of Piagetian quantity concepts is unclear. In the study just reviewed, Posner and Baroody found that a variety of counting skills were related to children's ability to understand number conservation, and Saxe (1979a, 1979b, 1981e) has demonstrated that the ability to use counting to produce numerical comparisons and reproductions of sets precedes the development of number conservation, both in the United States and in three traditional Papua New Guinea societies. Nonetheless, it is difficult to imagine how the enumeration of sets before and after a transformation could be the sole basis for the development of number conservation. Young children miscount to varying degrees, and as a consequence, could not derive an understanding of the logical necessity of conservation with reference to their counting alone (see Saxe, 1979a, for a general discussion of this issue).

The third theme in Piagetian research deals specifically with numeration

systems and the way that operational structures become interwoven with children's use and understanding of their numeration systems. In the few studies that bear on this problem, Saxe (1979b, 1981b, 1981c) has argued that wide variations in the organization of numeration systems may produce predictable kinds of conceptual difficulties for children who are in the process of acquiring them. In one study, Saxe (1981b) hypothesized that, due to the specific properties of the Oksapmin body part numeration system, Oksapmin children would experience conceptual confusions in understanding that each body part represents a distinct ordinal value in a progressive summation of values. Specifically, it was expected that they would evaluate the numerical relation between body parts, not on the basis of their ordinal position of occurrence, but on the basis of their physical similarity (i.e., equivalence of terms on right and left sides of the body). To test this hypothesis, Saxe required Oksapmin children to compare values of symmetrical body parts (e.g., the thumb on the right versus the left hand) and asymmetrical body parts (e.g., the right thumb versus the left elbow). The results support the hypothesis that, early in development, young children are seduced by the physical similarity of body parts in producing numerical evaluations on these tasks, but over the course of development, children come to use body parts to represent unique ordinal positions in a progressive summation of elements.

In another study conducted in Papua New Guinea (Ponam Island), Saxe (1981c) examined qualitative changes in children's understanding of an indigenous birth order naming system. In the Ponam birth order system (described earlier), names are assigned to children on the basis of both sex and birth order. The system has several interesting logical properties: For example, the relative age relation among individuals within a family can be determined on the basis of birth order names within a sex but not across sexes. On the basis of Piaget's theoretical formulation, Saxe reasoned that an understanding of the determinate age relations within sex should emerge with the advent of concrete operations since this understanding requires a subject to coordinate two series of asymmetrical relations (age and birth order name), a defining characteristic of concrete operational thought. It was also reasoned that an understanding of the indeterminate age relations across sexes would not emerge until the advent of formal operations since this understanding requires a subject to create a combinatorial system that, in principle, could generate all possible birth orders. Consistent with these hypotheses, the results showed that children acquire an understanding of the determinate relations before indeterminate relations and that an understanding of the indeterminate relations is not achieved until late adolescence. Both studies by Saxe represent attempts to incorporate an analysis of culture-specific knowledge systems within the framework of universal structural changes described by Piaget.

In summary, most studies inspired by the Piagetian approach succeed in documenting a shift from preoperational to concrete operational mathematical

concepts, whether these concepts are assessed with conventional tasks or those which make use of an indigenous knowledge system as a task context. Problematic for the Piagetian formulation is the lack of empirical support for the construct of stage, specifically, that there is a psychological reality to a content-free description of structural competence. In general, the cross-cultural literature, like the Western literature, shows that individuals display developmentally distinct forms of reasoning as a function of the type of assessment task that is administered. In addition, some researchers have not found evidence of either concrete or formal operational reasoning with particular types of assessment tasks in various cultural settings (see Dasen, 1972; Lancy, Souviney, & Kada, in press). We will briefly consider some of the attempts to reconcile these findings with Piaget's theory before we offer our own concluding remarks.

In an attempt to reconcile the lack of support for the the psychological reality of the construct of stage, Dasen (1977a, 1977b) employs a distinction between competence and performance used by other cognitive developmental theorists (see Flavell & Wohlwill, 1969). He proposes that the competence for thought structures at the concrete operational stage is universal, though its manifestation in a given situation may be culturally determined. In this way, if a subject manifests a concrete operational understanding on some but not all tasks, it is assumed that a performance factor is blocking the expression of the subject's competence. Although the distinction between competence and performance is a crucial one, Dasen's solution creates a serious paradox in evaluating the significance of cross-cultural findings. If Dasen's position is invoked post hoc by researchers attempting to deal with findings that contradict Piaget's theory, what will constitute negative findings with respect to the claim for universal thought structures? Priswerk (1976) has voiced a similar concern. In order to support his view, Dasen and others (Dasen, Lavallee, & Retschitzki, 1979; Dasen, Ngini, & Lavallee, 1979; Lavallee & Dasen, 1980) have used the training study as a means to reveal latent competence. If subjects do possess competence in some areas of concrete operational thought but, due to a performance factor, do not exhibit the competence in response to some task, then the competence should be expressed given a minimum amount of training. These types of training studies have met with mixed success.

In a more recent attempt to address the lack of empirical support for the construct of "stage" in cross cultural research, Harris and Heelas (1979) have proposed a hybrid model incorporating both contextualist and structuralist themes. They argue that operational structures are constructed by subjects in local conceptual contexts that are related to particular experiential factors. Unlike the classical Piagetian claim, Harris and Helas argue that these constructions do not generalize across conceptual domains but rather are linked to particular areas of experience.

Piaget (1972) has also considered the problem of the relation between cross-

cultural findings and this theoretical formulation and has adopted a position that embraces sociocultural context more closely than his earlier position. Concerning the acquisition of the formal operational stage during adolescence, he argues that individuals "reach this stage (formal operations) in different areas according to their aptitudes and their professional specializations (advanced studies or different types of apprenticeship for the various trades): The way in which these formal structures are used, however, is not necessarily the same in all cases [p. 10]." In fact, in this article, Piaget leaves open the issue as to whether the formal operational structures are an adequate characterization of the cognitive functioning of the adult and whether additional or alternative structures are yet to be uncovered.

Thus, it appears from this that Piaget himself has moved toward a greater relativism in considering endpoints of the cognitive developmental process. Certainly he has here, as he has elsewhere (1966), attempted to accommodate his theory to the results accumulated from cross-cultural research. Such attempts at accommodation notwithstanding, it should be reemphasized that although Piaget concedes that educational and cultural transmissions are contributing factors to cognitive development, the primary focus of his theory is on organism–environment equilibrations of a very general sort. These interactions result in a sequence of underlying structures that have universal features. The operational structures that are content-free inform intellectual activities that are enacted via culturally determined systems of knowledge. It is through this process that culturally defined systems of representation take on logical properties. The role that the cultural encodings or surface structure of particular concepts have on the development of cognitive structures has been largely considered unimportant and therefore ignored in the Piagetian tradition.

CONCLUDING REMARKS

In this chapter we have argued that from both Piaget's and Vygotsky's theoretical perspectives there are both universal and culture-specific processes that are entailed in the development of numerical cognition, although the nature of these processes vary across the two theories. Accordingly, the two bodies of research reviewed differ in the way in which researchers frame objectives, elaborate theoretical constructs, and interpret results. The research stemming from Piaget's perspective takes logical operations as the major focus of study. Researchers using this orientation have attempted to document universal changes in children's understanding of logical operations and to determine culture-specific factors that influence the rate of development as well as the representational contexts in which these operations are manifested. In contrast, those whose work is consistent with Vygotsky's approach have made the cultural context of prob-

lem solving the primary focus of study. Researchers in this tradition have attempted to gain a greater understanding of the general processes that foster culture-specific problem-solving strategies.

That each of these formulations has contributed to our understanding of the three core properties of numeration is undeniable. However, as we have pointed out, each approach lacks comprehensiveness, and where one theoretical approach tends to be strong, the other is weak. Piaget's account treats numeration systems as equivalent across contexts, focusing on the logical coordination of relations that must underlie the use of any system (Property 2), whereas Vygotsky's account focuses on the way in which the particular forms of number representation become interwoven with and influence the fabric of numerical thought (Properties 1 and 3).

We believe that a coordination of the objectives in the Vygotskiian and Piagetian formulations can be achieved that would give rise to more comprehensive research programs, programs that could lead to an understanding of the interrelatedness and interactions between the formation of Properties 1, 2, and 3. In a new series of studies among the Oksapmin, one of us has employed such a coordinated approach (see Saxe, 1981d, 1981e, 1982, in press). We will review only a single study here as an example (Saxe, in press).

Recall that the Oksapmin use a body part counting system. In traditional Oksapmin life, there is virtually no social context in which one needs to carry out arithmetic computations. However, in recent years, a money economy has been introduced in the region and this new economy has created a new social context within which individuals must create solutions to addition and subtraction problems involving currency. It was expected that this new social context would lead Oksapmin to construct novel ways of using their body system to mediate arithmetical computations, a constructive process involving each of the core properties of numeration. First, it was expected that traditional forms of number representation would be ill-suited to handle manipulations with currency; however, with the introduction of the money economy, it was expected that Oksapmin would transform the traditional "material" in such a way as to achieve their new problem-solving ends involving the addition and subtraction of currency (Property 1). Second, it was expected that this transformation would be associated with underlying structural changes in the logical properties of numerical operations (Property 2). Finally, the new forms of problem-solving strategies should represent general procedures for the conceptual manipulation of numerical relations (Property 3). The way in which this change in the organization of number representation was studied was to interview individuals with varying experience with the money economy; these included four groups ranging from tradestore owners to traditional adults who had little contact with the money economy. Each individual was asked to solve arithmetic problems involving currency.

An analysis of peoples' strategies showed differences in each of the core

properties as a function of the amount of participation in the money economy. Traditional individuals typically considered only correspondence relations between a series of body parts and a set of real-world objects in their solution procedures. For example, to subtract 9 from 16, individuals with little experience would typically enumerate the corresponding series of body parts as an answer—for example, shoulder (10), neck (11), ear (12), eye (13), nose (14), eye on other side (15), and ear on other side (16). In other words, they used each body part to represent a coin and then simply iterated the body parts that would be left once the transaction was completed, without offering a single numeric representation for the answer. Similarly, to add 7 + 9 without coins present, an individual with little experience would typically enumerate the body parts—shoulder (10), neck (11), ear (12)—and simply estimate when the enumeration should be ended since they would not be able to tell when they had completed enumerating seven additional body parts. In contrast, people who had regular experience with economic exchange created new forms of correspondence operations that enabled them to keep track of the addition or subtraction of coins. These individuals would often establish correspondences between a series and a subseries of body parts so that they achieved an exact representation of the answer by keeping a running record of the progressive addition or subtraction of body parts. For instance, to subtract 9 from 16 without coins, a typical strategy would be to "count-on" from the shoulder (10) to the ear-on-the-other-side (16) by using a subseries of the terms: thumb (1), index finger (2), middle finger (3), . . . forearm (7). Thus, these people would achieve a precise numerical representation ("forearm" or 7) for the product of the subtraction. Similarly, to add 7 + 9, an individual would call the shoulder (10), the thumb (1), the neck (11), the index finger (2), the ear (12), the middle finger (3), and so forth until a correspondence was reached between the ear-on-the-other-side (16) and the forearm (7).

An inherent feature of the transformation in number representation among the Oksapmin is a change in each of the core properties. First, Oksapmin are reorganizing their traditional "material" of number representation (the body system) to solve more adequately numerical problems that arise with participation in the money economy (Property 1). For instance, body parts are shifting in their function from being signifiers of solely real-world objects to means of representing numerical values in the addition or subtraction of values. Moreover, the transformation is inherently linked to Oksapmins' elaboration of logico-mathematical structures of correspondence (Property 2), as discussed in the "counting-on" strategy. For instance, correspondence operations are now constructed between a series and subseries of body parts in order to achieve problem-solving ends. Finally, though the novel strategies are being elaborated in the context of particular exchanges with currency, the strategies are general ones that

the Oksapmin use to mediate their thinking about the addition and subtraction of currency (Property 3).

The conclusion of the present review is that the formation of mathematical concepts is a developmental process simultaneously rooted in the constructive activities of the individual and in social life. If we are to devise comprehensive models of number development, we must build into these models an account of the way these factors are interwoven with one another in the developmental process. To date, our conceptual models and empirical research have not systematically addressed this fundamental concern. Either the cultural context of problem solving is the focus of study and research does not inform our understanding of structural developmental change, or culture is unanalyzed in cross-cultural research, and the focus is on the universal development of cognitive structures. It is our expectation that future work will present fresh approaches.

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