The Mathematics of Child Street Vendors

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SAXE, GEOFFREY B. *The Mathematics of Child Street Vendors*. CHILD DEVELOPMENT, 1988, **59**, 1415–1425. The mathematical understandings of 23 10–12-year-old candy sellers with little or no schooling from Brazil's northeast were compared to 2 groups of nonvendors matched for age and schooling—a group from the same urban setting and a group from a nearby rural setting. Children's performances were analyzed on 3 types of mathematical problems: representation of large numerical values, arithmetical operations on currency values, and ratio comparisons. Vendors and nonvendors alike had developed nonstandard means to represent large numerical values, an expected result since problems involving large values emerge in the everyday activities of each population group. Most vendors, in contrast to nonvendors, had developed adequate strategies to solve arithmetical and ratio problems involving large numerical values, also an expected finding since these problem types emerge frequently only in the everyday activities of the vendor population. The findings are interpreted as supporting a model of cognitive development in which children construct novel understandings as they address problems that emerge in their everyday cultural practices.

The present study is an investigation of the influence of cultural practices on the cognitive development of children with little or no schooling. The focus is the influence on children's mathematical understandings of participation in the street vending practice, a common activity for unschooled children in developing countries. The population targeted for study were 10–12-year-old candy sellers who come from a poor urban community in Brazil's northeast.

Research on mathematical understandings has increasingly documented a wide array of competences that Western children acquire outside the context of formal instruction. Very young children know principles underlying counting (Gelman & Gallistel, 1978; Saxe, Guberman, & Gearhart, 1987; Wilkinson, 1984); by early elementary school, children show the ability to solve Piagetian measures of numerical understanding, such as number conservation (Piaget, 1952), and the ability to use a variety of arithmetical problem-solving procedures that are not directly taught in school (Brown & Burton, 1978; Fuson, 1982; Ginsburg, 1977; Groen & Parkman, 1972; Resnick, 1982). Children's informal mathematical knowledge often is initially limited to small sets and progressively extended to larger sets (Gelman, 1972; Siegler & Robinson, 1982; Winer, 1974), and, in solving numerical problems for any set size, children's confidence in the correctness of their answer often influences the choice of a particular problem-solving strategy (Siegler & Shrager, 1984).

Research with both non-Western populations and populations in developing countries where schooling is not universal points to the wide variations that mathematical procedures can take (see Menninger, 1969; Saxe & Posner, 1983; and Zaslavsky, 1973). For instance, in solving a computational problem that may occur in the marketplace, such as 37 + 24, unschooled individuals typically do not use solutions that involve the standard carrying algorithm in which the unit values are added, a unit of 10 units is carried, and then the units of 10s values are added (Carraher, Carraher, & Schliemann, 1985; Ginsburg, Posner, & Russell, 1981; Pettito, 1979). Rather, they will often restructure the problem into "convenient" values for which they know the sums, proceeding, in contrast to the formal algorithmic procedure, from 10s to unit values. Using such a procedure, 37 + 24 may become

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30 + 20 and 7 + 4, which in turn may become 50 + 10 + 1, and in turn yields the sum of 61. The developmental course whereby individuals acquire such alternative procedures is not clear, though comparative research indicates that such achievements are linked to the everyday practices in which individuals participate (Greenfield & Childs, 1977; Posner, 1982; Price-Williams, Gordon, & Ramirez, 1969).

The view adopted here is that the particular form that informal mathematical knowledge takes is the result of an interplay between the character of the mathematical problems with which individuals are engaged in everyday practices and the prior knowledge that they bring to bear on those problems. Evidence supporting this conceptualization comes from the few comparative studies of cognitive development that have documented indigenous knowledge forms such as the numeration system of a cultural group and practice-linked mathematical problems (Brenner, 1985; Reed & Lave, 1979; Saxe, 1982, 1985). Saxe (1982), for example, documented the indigenous mathematics of the Oksapmin, a remote group from Papua New Guinea who use a nonwritten 27-body-part counting system to represent number in traditional activities (such as counting valuables). In their traditional mathematics, Oksapmin rarely if ever produce arithmetical operations. Recently, a money economy was introduced into the area, and any Oksapmin who engaged in commercial transactions had to deal with these novel problem forms. These Oksapmin structured arithmetical operations that were based on the body-part structure of their own system, and the level of sophistication of their body-part-linked operations was associated with the extent of experience with the new money economy.

In the present research, Brazilian candy sellers were selected for study because of the unique nature of the knowledge they bring to bear on their practice as well as the complexity of the mathematical problems with which they are engaged. A long history of monetary inflation in Brazil has resulted in highly inflated prices. As a result, all Brazilian children identify and use large numerical values as a part of everyday activities such as running family errands to the local stores and vegetable markets or purchasing popcorn and candy for themselves (Guberman, 1987). Thus, upon entry into the selling practice, children already have some prior knowledge of large numerical values and some means of representing these values, perhaps by associating figurative characteristics of bills with linguistic numerical values.

In the candy selling practice, more complex mathematical problems involving arithmetic with multiple bills and ratio comparisons emerge, problems that engage children's prior knowledge of large numbers and their ability to represent large numerical values. Sellers must buy boxes containing 30 to 100 units (e.g., rolls of Lifesavers, candy bars) from one of many wholesale stores, price the units for retail sale, sell their merchandise to customers in the street (typically poor to middle-class people riding on the bus or walking in the downtown area), and then prepare to purchase a new box (which ranged in value from Cr$3,500 to Cr$20,000 during the period of the study).[1] Problems requiring adding and subtracting multiple bills of large values emerge when sellers must determine whether they have enough cash to purchase a wholesale box of candy or when sellers make an effort to keep track of the amount of cash they have during the course of their sales. Ratio comparisons emerge as a result of a pricing convention that has evolved to reduce the complexity of the arithmetical problems in retail sales transactions. Sellers offer their merchandise to customers in multiple units of candy for a given bill denomination (e.g., X units for 1,000 cruzeiros, Cr$1,000). Though this convention has the effect of simplifying the arithmetic of retail sales transactions, it introduces ratio-comparison problems when sellers consider the relative merits of different retail prices (such as 5 for Cr$1,000 vs. 2 for Cr$500).

In this study, the primary goal was to understand both the nature of children's prior knowledge of large-number representation and the emergence of mathematical understandings as a function of practice participation. To this end, sellers' understandings and solution strategies were contrasted with those of children with less experience in addressing

[1] Rarely is the candy selling occupation the exclusive year-round practice of a child candy seller. Rather, depending upon children's perceptions of market conditions, children shift their selling practices to such merchandise as fruit (especially for seasonal crops of specific fruit types), popsicles (especially during the hot summer months), puffed wheat (especially when wholesale prices are favorable relative to other merchandise), or plastic shopping bags (especially at Christmas time).

problems that arise in the selling practice—an urban and a rural group of nonsellers—on tasks involving number representation, arithmetic problem solving, and ratio comparison. Urban nonsellers are engaged as buyers with generally the same commercial environment as the sellers, but not as vendors. Rural nonsellers use the same currency system, but their level of exposure to commercial transactions is considerably more limited than that of their urban counterparts. Thus, contrasts across groups should reveal whether engagement with practice-linked mathematical problems leads children to construct differing forms of mathematical understandings.

There were two additional secondary goals of the research that bear on current work in the development of mathematical cognition. First, Brazilian children's use of large as contrasted with small currency values in everyday activities may lead them to greater facility with the larger values in numerical comparisons; such a finding would add a complicating factor to models of number development in which increases in set size are related to poorer performances on numerical tasks (e.g., Siegler & Robinson, 1982). Second, Siegler and Shrager's recent work on strategy choice suggests that the urban Brazilian children's arithmetical problem-solving strategies in everyday bill calculations—problems that involve adding multiple bills of multiple denominations—may vary as a function of problem complexity. Therefore, bill addition problems were used that varied in the number and variety of denominations to be summed.

Method

Subjects

All sellers were recruited by Brazilian university students in the streets as the sellers sold candy, tangerines, or puffed wheat. Children who were vending in the streets were administered a preliminary screening interview to determine their candy selling experience: a procedure was devised to determine a minimum length of time that sellers had participated in the candy selling practice by asking whether they had been selling candy at one of several specified Brazilian holidays, and whether they were still engaged with the candy selling practice by the time the next holiday occurred. Sellers were also questioned about their age and their schooling experience. Sellers were included in this study only if they had sold candy for at least a 3-month period (e.g., from one holiday to another), were between 10 and 12 years of age, and had not progressed beyond the second grade. Sellers were never told the bases for their selection.

The urban nonsellers included in the study were recruited from first- and second-grade classes at public schools in Recife, the same or similar schools that sellers attended, had attended, or would have attended if they had enrolled in school. Potential subjects were administered an interview to determine whether they had any vending experience, and only children who did not were selected to participate in the study. Like the sellers, all urban nonsellers were between 10 and 12 years of age. Unlike the sellers, all urban nonsellers were enrolled in school during the time the study was conducted.

The rural nonsellers were 10–12-year-olds recruited in small remote towns about 100 miles from Recife, which were our sites for an ethnographic study on straw weaving.[2] Like the urban children, the rural children were included only if they had not progressed beyond the second grade.

The complete sample for the present study included 23 sellers (mean age = 10.8 years, SD = 1.0 year; mean grade level = 1.6, SD = 0.6), 20 urban nonsellers (mean age = 10.8 years, SD = 0.8 years; mean grade level = 1.8, SD = 0.4), and 17 rural nonsellers (mean age = 10.6 years, SD = 0.8 years; mean grade level = 1.3, SD = 0.5).

Procedures

The nonsellers were presented with a description of the selling practice during the initial phase of the interview and told that they were to pretend that they were a candy seller during the interview. Even though children did not sell candy in the rural community, a couple of children did occasionally sell popsicles, and therefore the rural children were somewhat familiar with the idea of child vendors (rural sellers were not included in this sample). Many rural children also made occasional trips to towns where they had an opportunity to observe vendors. Thus, despite the relative lack of experience with vending practices, rural children had some exposure to such practices with which to interpret our description of the candy selling practice. Further, to facilitate children's understanding of

[2] In a separate study on spatial cognition in which rural children from this community were contrasted with urban candy sellers on topological tasks, the rural children generally demonstrated superior performances to the candy sellers.

the tasks, the nonsellers were told the function of each task in the selling practice.

Identifying and Comparing Numerical Values

Three tasks described below were used to assess children's ability to identify and compare large numerical values. In the first, children's ability to identify and compare values using the standard number orthography was assessed. In the two remaining tasks, children's ability to identify and compare large numerical values using figurative characteristics of bills as contrasted with the standard orthography was assessed.

Identifying numerical values using the standard orthography.—Children were presented with 10 pairs of numbers written in the standard orthography, pair by pair, and asked to read and compare them; if children did not attempt to read the numbers aloud (most did), they were asked to attempt to read them again. Each number of the pair was printed on a card (about 10 × 10 cm). Five number pairs were embedded in a question about cruzeiros, and five pairs were embedded in a question about the number of candies in a sack (a familiar means of packaging candy). The order of question type for each block of five pairs was counterbalanced, and one-half of the subjects within each age group were administered set A number pairs (2,000 vs. 1,500; 3,456 vs. 4,000; 935 vs. 1,000; 150 vs. 146; 168 vs. 321) for the cruzeiro questions and set B (3,000 vs. 2,500; 4,567 vs. 5,000; 847 vs. 1,000; 160 vs. 152; 489 vs. 932) for the candy questions, and the other half were administered set B for the cruzeiro questions and set A for the candy questions.

Identifying numerical values on the basis of the orthography versus figurative characteristics of bills.—On the basis of ethnographic work with the candy sellers (Saxe, in preparation), it appeared that sellers might be using the figurative characteristics of bills as bases for their arithmetical computations. To assess children's ability to determine the numerical value of bills by relying on the bills' figurative characteristics, as contrasted with the printed orthography, children were presented with 12 trials requiring them to identify numerical values in each of three conditions: one in which they had to identify the numerical values of bills; another in which they had to identify the numerical values of identical bills, but with the printed numbers on the bills occluded with tape; and another in which children had to identify the printed number on the bills without referring to the bills' figurative characteristics (the numbers on the bills were photocopied, and the photocopied numbers were used as stimuli). The bill values, two each of six denominations—Cr$100, Cr$200, Cr$500, Cr$1,000, Cr$5,000, Cr$10,000—were the same for each condition. The trials were randomized for each subject, and the conditions were counterbalanced within each population group. In the Standard Bills and Number Occluded conditions, the interviewer said, "Now I'm going to show you some bills and you tell me how much each is worth," and children were then presented with each bill individually. In the Numbers Only condition, the interviewer told the child, "Now I'm going to show you some numbers from bills and you tell me how much each is worth."

Identifying numerical relations between currency values.—The purpose of this task was to assess children's knowledge of the structure of the currency system. Children were presented with 14 pairs of bills and/or coins pair by pair (the order of presentation was randomized for each subject) and asked to compare them to determine which was the greater value and how many of the lesser value were equivalent to the greater value. Seven of these pairs involved values of Cr$100 or less (e.g., Cr$10 vs. Cr$50), and seven involved comparisons involving values of Cr$100 or more (e.g., Cr$1,000 vs. Cr$5,000). With the presentation of each pair, the interviewer said, "Which of these is more money?" [Child responds.] "Let's say that you have this money [the greater value] and you go to a store to get change. How many coins (or bills) of [the lesser value] do you need to get for change?"

Arithmetic: Addition and Subtraction of Large Values

To assess children's ability to solve addition problems involving multiple bill values, children were presented with two addition problems that differed in the number of bills to be added and the sum of those bills. In each problem, the child was presented with a stack of bills in a standard but haphazard order. In the first problem, the child was presented with 12 bills (totaling Cr$8,600) and told, "Suppose you started the day with this amount of money [the interviewer handed the child the stack]. Would you add the money for me?" The child was provided with a paper and pencil and told that he could use the paper and pencil, talk aloud, or anything else he liked to help him find out how much was in the stack. In the second addition problem, the child was presented with 17 bills (totaling Cr$17,300). Because problems involving such large quantities of bills were unusual for the rural children's experience, the rural children were only administered the first of the two problems.

To assess children's ability to solve subtraction problems, children were presented with two problems that differed only in the price of a box to be purchased and the amount of money available to purchase the box. In one problem, children were told, "Suppose you decided to buy this box of Chupetim [a box of Chupetim candy was presented] that costs Cr$3,800 in the store. Show me how much you would give to the clerk." This task followed the addition problem totaling Cr$8,600, and the child had the stack of Cr$8,600 from which he could offer payment. If the child did not offer the one Cr$5,000 bill from the stack, the child's value was noted, and the child was presented with the Cr$5,000 bill and told that he should give the clerk that amount. The child was then asked, "Should you receive change?" If the child responded yes, the following additional questions were asked: "How much change? How do you know? Show me." In the second subtraction problem, children were told, "Suppose you decided to buy this box of Danubio that costs Cr$7,600 in the store [a box of Danubio candy was presented]. Show me how much you would give to the clerk." This task followed the addition problem totaling Cr$17,300, and the child had the stack of Cr$17,300 from which he could offer payment. If the child did not offer the one Cr$10,000 bill from the stack, the child's value was noted, and the child was presented with the Cr$10,000 bill and told that he should give the clerk that amount. The remainder of this task was identical to the first subtraction task.

Ratio Comparisons

To introduce the ratio-comparison problems, the interviewer presented a bag of Pirulitos (another type of candy) and told the child the following: "Suppose that you bought this bag of Pirulitos, and you must decide the price you will sell the pieces for in the street." The child was then administered three problems successively that varied in pricing ratios to be compared: 1 Pirulito for Cr$200 versus 3 Pirulitos for Cr$500, 1 Pirulito for Cr$200 versus 7 Pirulitos for Cr$1,000, 3 Pirulitos for Cr$500 versus 7 Pirulitos for Cr$1,000. For each of the three problems, the following problem format was used (illustrated for the first problem): "Let's say that you have to choose between two ways of selling: selling 1 Pirulito for Cr$200 or selling 3 Pirulitos for Cr$500 (1 Pirulito was placed next to a Cr$200 bill and 3 Pirulitos were placed next to a Cr$500 bill). Which way do you think that you would make the most profit? Why? How do you know you would make a bigger profit this way? Show me what you did to discover that you have a bigger profit this way." If the child did not demonstrate his strategy for solving the problem, he was presented with the following countersuggestion: "Another boy that I talked with chose [the opposite of the subject's choice] as the one that brought more profit. I want you to show me how you would prove to this child that he was wrong."

Results

Children's Knowledge of the Standard Orthography

Identifying numerical values using the standard orthography.—Children's recognitions of each of the 20 numbers were coded with respect to one of three categories. These consisted of correct recognition, and one of two error codes: partitioning number strings into smaller segments (e.g., 129 read as 1, 2, and 9), and place value errors involving the addition or subtraction of zeros (e.g., 129 read as 1,029). Partitioning errors suggest that children do not have rules for incorporating a series of digits into a single numerical expression. Place value errors suggest a more specific problem in appropriately accommodating multidigit numbers to place value rules. With this scheme, it was possible to code 67% of all responses. The predominant response types uncoded with this scheme were responses in which the child stated that he did not know the answer or responded with a number that had no apparent relation to the targeted number.

The proportion of each child's responses for each category was determined, and the means of these proportions as a function of problem condition (currency vs. candy problems) and population group (sellers, urban nonsellers, rural nonsellers) are presented in Table 1. The table shows that none of the population groups displayed a high level of competence in reading numbers. Indeed, the urban nonsellers, the group with the greatest mean proportion correct, only attained on the average one-half of the number correct. A 3 (population group) × 2 (problem condition, repeated) ANOVA on proportion of correct responses revealed an effect only for population group, $F(2,59) = 12.83$, $p < .0001$. Duncan's multiple-range test revealed that the urban nonsellers achieved more correct re-

TABLE 1

MEAN PROPORTIONS OF NUMERAL RECOGNITION RESPONSE TYPES AS A FUNCTION OF POPULATION GROUP

AGE GROUP	Correct	Add/Subtract Zeros	Partitions
Sellers:			
Mean	.39	.29	.04
SD	.30	.19	.11
Nonsellers (urban):			
Mean	.51	.28	.02
SD	.25	.17	.07
Nonsellers (rural):			
Mean	.10	.11	.24
SD	.22	.15	.29

sponses than the sellers, and the sellers achieved more correct responses than the rural nonsellers (p's $< .05$).

Two additional 3 (population group) × 2 (problem condition, repeated) ANOVAs were performed to determine differences in children's error patterns. The analyses revealed effects only for population group in each analysis: proportion of partitioning errors, $F(2,59) = 8.93$, $p < .0005$; proportion of place value errors, $F(2,59) = 6.95$, $p < .005$. Duncan multiple-range tests (collapsing across problem conditions) revealed that the rural nonsellers committed the less sophisticated error type—partitioning errors—to a greater extent than both the sellers and urban nonsellers, and that the sellers and urban nonsellers committed the more sophisticated error type—place value errors—to a greater extent than the rural nonsellers.

Children's ability to produce ordinal comparisons between the numbers was analyzed by assigning 1 point for each of the 10 comparisons. Because the probability of attaining a correct answer for each comparison by random responding is .5, a binomial expansion was used to establish a criterion of at least 8 points for "passing" (the probability for obtaining a score of 8 or greater by random responding is .055). The majority of the sellers (70%) and the urban nonsellers (75%) achieved passing scores, whereas only a minority of the rural nonsellers achieved passing scores (21%). Significantly fewer rural nonsellers produced passing comparison scores than did sellers, $\chi^2(1, N = 42) = 7.96$, $p < .005$, and urban nonsellers, $\chi^2(1, N = 39) = 9.29$, $p < .005$.

In sum, as expected, children's knowledge of the standard orthography is quite limited. All groups performed poorly at identifying values, and although the majority of the urban groups achieved significantly better than chance scores in comparing numerical values, by itself the utility of such an ability for producing everyday calculations and price evaluations is quite limited. An unexpected finding in identifying numerical values was the superior performance of the urban nonsellers to the sellers. In retrospect, the likely source of difference is a greater recency of the nonsellers' school experience. All of the nonsellers were enrolled in first or second grade at the time of the study. In contrast, many of the sellers either were not currently enrolled in school or were never enrolled.

Identifying numerical values on the basis of the orthography versus figurative characteristics of bills.—To analyze the bases upon which children identify the numerical values of bills, the number of correct identifications of the 12 values in each condition was summed for each child. Table 2 contains the mean number of correct identifications as a function of condition (Standard Bills, Numbers Occluded, and Numbers Only) and population group. A 3 (condition) × 3 (population group) ANOVA revealed main effects for both condition, $F(2,120) = 80.83$, $p < .0001$, and population group, $F(2,60) = 9.37$, $p < .0005$, and a group condition interaction, $F(4,120) = 9.72$, $p < .0001$. Duncan multiple-range tests showed that for the Standard Bills condition, there were no group differences (virtually all children from each group achieved all correct); however, for the Numbers Occluded condition, sellers and urban nonsellers correctly identified more bills than the rural nonsellers, and for the Numbers Only condition (consistent with the urban nonsellers' performance on the previ-

TABLE 2

MEAN NUMBER CORRECT ON ALTERNATIVE NUMBER-REPRESENTATION TASK AS A FUNCTION OF CONDITION AND POPULATION GROUP

POPULATION GROUP	STANDARD BILLS		NUMBERS OCCLUDED		NUMBERS ONLY	
	Mean	SD	Mean	SD	Mean	SD
Sellers	11.54	2.30	11.96	.27	7.62	3.60
Urban nonsellers	12.00	.0	11.90	.31	10.30	2.03
Rural nonsellers	11.88	.49	11.53	.80	4.88	3.66

NOTE.—Maximum possible score is 12.

ous orthography task), urban nonsellers correctly identified more number values than sellers and sellers identified more than rural nonsellers. Duncan multiple-range tests also revealed a consistent pattern in performance across groups: no groups differed in their performance on the Standard Bill condition as contrasted with the Numbers Occluded condition, and all groups performed better on the Standard Bills and Numbers Occluded conditions as contrasted with the Numbers Only condition.

These results, then, do indicate that children across population groups had developed an ability to use bills themselves as signifiers for large values and did not need to rely on their imperfect knowledge of the standard number orthography to represent and identify large values.

Children's Knowledge of Numerical Relations between Currency Values

Though the previous analysis indicates that children are quite able to identify the numerical values of currency, these analyses do not indicate whether children treat these values as terms in a numerically ordered system. Such an analysis would require information on children's understanding of numerical relations between the bill values. Two aspects of children's ability to identify numerical relations between currency values were analyzed: knowledge of ordinal relations between currency units and knowledge of numerical relations between currency units.

Ordinal relations.—Children's ability to produce ordinal comparisons between currency units was analyzed by assigning one point for each of the seven comparisons for each problem type. Like the orthography-based comparisons, because the probability of attaining a correct score for each comparison by random responding was .5, a criterion of at least 6 points was set as "passing" (the probability of obtaining a score of at least 6 by random responding is 0.062). The large majority of all groups produced passing scores on comparisons involving values of Cr$100 and less (sellers: 87%; urban nonsellers: 85%; rural nonsellers: 94%) and Cr$100 and more (sellers: 96%; urban nonsellers: 85%; rural nonsellers: 100%). With so little variation in performance, a 2 (currency values) × 3 (population group) ANOVA revealed no significant effects.

Cardinal relations.—Table 3 contains the mean number of correct answers to questions about the numerical relations between currency units as a function of currency values (Cr$100 and below vs. Cr$100 and above) and population group. Like the children's ability to identify values on the basis of bills' figurative characteristics, children generally achieved high levels of performance on these conditions, particularly the comparisons involving large values. A 2 (currency value) × 3 (population group) ANOVA revealed a significant interaction between currency value level and population group, $F(2,57) = 3.88$, $p < .05$, and a main effect for condition, $F(1,57) = 24.42$, $p < .001$. Duncan comparisons ($p < .05$) revealed that both the urban and rural nonsellers achieved more correct answers to the problems involving the larger as contrasted to the smaller values, whereas there was no difference in the sellers' performance across the problems. Duncan multiple-range tests also revealed that the sellers achieved more correct identifications on the smaller-value problems than the rural nonsellers ($p < .05$).

Children's Arithmetic

Adding currency values.—Children's solutions were scored as the absolute difference between each child's summation and the actual value of the bills (Cr$8,600 in problem 1 and Cr$17,300 in problem 2), and then these values were assigned to one of three ordered categories presented in Table 4: correct, off by no more than Cr$200, and off by more than Cr$200. Kruskal-Wallis one-way ANOVAs

TABLE 3

NUMBER OF CORRECT IDENTIFICATIONS OF MULTIPLICATIVE RELATIONS BETWEEN CURRENCY UNITS AS A FUNCTION OF CURRENCY VALUE AND POPULATION GROUP

POPULATION GROUP	CURRENCY VALUES: Cr$1 to Cr$100 Mean	SD	Cr$100 to Cr$10,000 Mean	SD
Sellers	6.30	1.11	6.74	.54
Urban nonsellers	5.90	1.89	6.70	.57
Rural nonsellers	4.82	2.01	6.65	1.00

NOTE.—Maximum possible score is 7.

were performed on children's category scores. These analyses revealed a significant difference, as a function of population group on problem 1, $\chi^2(2, N = 58) = 11.14, p < .005$; sellers achieved more accurate sums than both urban nonsellers (Mann-Whitney *U* test, $U = 141, z = -2.18, p < .05$) and rural nonsellers (Mann-Whitney *U* test, $U = 83, z = -3.35, p < .001$). Though a larger percentage of sellers achieved correct summations than did nonsellers on problem 2, this difference did not attain statistical significance (Mann-Whitney *U* Test, $U = 172.5, z = 0.77$). Consistent with the prior findings of children's poor performance on the standard orthography tasks, none of the children used the number orthography to represent the problem in paper-and-pencil solution strategies.

Despite the fact that problem 2 is the more complex of the two problems, a Wilcoxon matched-pairs signed-ranks test revealed no difference in accuracy as a function of problem condition for either sellers or urban nonsellers. An analysis of children's strategies across problems suggests a reason for the lack of a problem effect: some of the children shifted the order of arrangement of bills to create subgroups that could be summed to convenient values—multiples of Cr$500 or Cr$1,000. Shifts tended to occur more frequently for the more difficult problem for both the sellers (50% vs. 18%) and the urban nonsellers (45% vs. 35%). Thus, a plausible reason for the lack of problem difference was children's decision to invoke convenient value-rearrangement strategies on the more difficult of the two problems, a phenomenon consistent with Siegler and Shrager's (1984) analysis of strategy choice.

Children's subtractions.—Like the addition problems, children's solutions to the subtraction tasks were scored as the absolute difference between each child's answer and the correct answer to the subtraction problem; these absolute values were then assigned to one of three ordered categories presented in Table 5: correct, off by no more than Cr$200, and off by more than Cr$200. A Kruskal-Wallis ANOVA of children's category scores as a function of population group revealed a significant effect for problem 1, $\chi^2(2, N = 59) = 8.39, p < .05$, and Mann-Whitney *U* tests revealed that rural nonsellers achieved significantly less accurate scores than both sellers ($U = 104, z = 2.89, p < .05$) and urban nonsellers ($U = 118, z = 1.72, p < .05$). On problem 2, urban nonsellers achieved less accurate scores than sellers: Kruskal-Wallis, $\chi^2(1, N = 41) = 6.5, p < .05$. Though virtually all sellers and urban nonsellers offered the appropriate amount in the initial purchase requests in problem 1—the Cr$5,000 bill (86% and 100%, respectively)—significantly fewer nonsellers offered the appropriate amount

TABLE 4

PERCENT DISTRIBUTION OF ACCURACY SCORES TO CURRENCY-SUMMATION PROBLEMS

POPULATION GROUP	PROBLEM 1 Correct	+/−200	>200	PROBLEM 2 Correct	+/−200	>200
Sellers	82	9	9	59	9	32
Urban nonsellers	53	5	42	44	17	39
Rural nonsellers	29	18	53	...	...	...

TABLE 5

PERCENT CORRECT ON SUBTRACTION PROBLEMS AS A FUNCTION OF POPULATION GROUP

POPULATION GROUP	PROBLEM 1			PROBLEM 2		
	Correct	+/−200	>200	Correct	+/−200	>200
Sellers	61	0	39	65	0	35
Urban nonsellers	42	0	58	22	6	72
Rural nonsellers	12	6	82	...	...	...

(53%), $\chi^2(2, N = 58) = 13.58, p < .01$. Virtually all sellers and urban nonsellers offered the appropriate denomination, Cr$10,000, for problem 2 (91% and 95%, respectively). Like the addition problems, children did not use paper-and-pencil solution strategies for the subtraction problems.

Ratio Problems

Two aspects of children's solutions to the ratio problems were analyzed: children's accuracy in their judgments about which ratio would provide the greatest profit and their use of appropriate justifications. Table 6 contains both the percent of children who made the appropriate ratio judgments and the percent of children who offered appropriate justifications—justifications in which a child made reference to a calculation based on a common term (e.g., 1 for Cr$200 vs. 3 for Cr$500 transformed to a comparison of either 3 for Cr$600 vs. 3 for Cr$500 or 5 for Cr$1,000 vs. 6 for Cr$1,000). Of the incorrect judgments, 75% were accompanied by justifications that made reference to the absolute value to be gained in the exchange. Of those children who did produce an appropriate judgment and justification, a large percentage of children also spontaneously indicated the amount of additional profit (in candy, cruzeiros, or both) that they would make with their appropriate choice. Such appropriate analyses of profit accompanied 47% of the common term justifications.

To produce a single index of children's performance across problems for their judgments and for their justifications, children were assigned 1 point for each adequate judgment and 1 point for each adequate justification. A judgment and justification score thus each ranged from 0 to 3 for each child. ANOVAs on judgment and justification scores revealed a significant effect for population group: judgment: $F(2,57) = 7.57, p < .005$; justification: $F(2,57) = 10.05, p < .0005$. Duncan multiple-range tests ($p < .05$) revealed that for each analysis, sellers achieved more appropriate responses than both groups of nonsellers, and that there were no significant differences between urban and rural nonsellers' performances.

Discussion

The primary goal of this study was to document the interplay between children's developing mathematical understandings and the everyday mathematical problems with which they are engaged. I noted that from an early age poor Brazilian children address mathematical problems when they use currency in their everyday lives in such activities as purchasing a grocery item at a store. Because of the inflated currency, these activities give rise to the need to represent large numerical values and—to a limited extent—arithmetical calculations involving large values. Eventually, some urban children take

TABLE 6

PERCENT DISTRIBUTION OF CHILDREN'S APPROPRIATE JUDGMENT AND JUSTIFICATION RESPONSES AS A FUNCTION OF POPULATION GROUP

POPULATION GROUP	RATIO COMPARISON					
	1/200 vs. 3/500		1/200 vs. 7/1,000		3/500 vs. 1,000	
	Judgment	Justification	Judgment	Justification	Judgment	Justification
Sellers	78	70	78	74	83	78
Urban nonsellers	40	35	50	40	45	25
Rural nonsellers	18	12	35	24	35	24

up "street professions" such as candy selling, and in the context of this practice, mathematical problems of everyday life increase in complexity. Candy sellers must produce frequent computations involving multiple bills as well as compute and compare pricing ratios.

Despite the need to represent large values in their everyday lives, children, regardless of population group, did poorly on tasks requiring them to read multidigit numerical values. Both the partitioning and the place-value errors indicate that many children had not acquired the conventional rules for composing digits into multidigit values, even though they used appropriate linguistic descriptors (e.g., "5,000 cruzeiros") in their everyday activities. Despite problems with the standard orthography, children did well on the tasks requiring them to identify and compare numerically currency values on the basis of the figurative characteristics of bills and coins. These performances provide a clear indication that unschooled Brazilian children had available a representational means for number that was distinct from the standard orthography.

Children's ability to use bill values in arithmetical calculations was related to the nature of their everyday practices. For both the addition and subtraction tasks, the performances of the urban children were more adequate than those of the rural children, a result that points to the influence of participation in commercial transactions on children's developing procedures to accomplish arithmetic on bill values. The more adequate performances of the urban sellers as contrasted with the urban nonsellers on the subtraction problems points to the specific contribution of candy selling practice participation on children's developing mathematical knowledge.

Ratio problems are generally limited to the everyday activities only of the candy sellers, and the findings on the ratio-comparison tasks indicated, as expected, that sellers had generated more sophisticated understandings than the other two groups. The majority of the sellers not only accurately identified the pricing ratios that would provide the greater profit but also explained the rationale guiding these choices as one that involved generating a common term to compare ratios. In contrast, relatively few nonsellers produced comparable judgments and justifications.

In addition to the primary goal, the findings of the study also bear on current issues regarding the influence of set size as well as the relation of problem complexity to strategy choice in children's mathematical problem solving. Both the urban and rural nonsellers found the small-number comparisons involving currency units more difficult than the large-number comparisons, a finding that indicates that the functional utility of bill value magnitudes plays a role in children's knowledge about numerical relations, and one that adds a complicating factor to the well-documented effects of numerical magnitude on quantitative operations (see Siegler & Robinson, 1982). The Brazilian children's strategy choices on the multibill addition problems were in accord with recent work in the United States on strategy choice (Siegler & Shrager, 1984), research indicating that children shift strategies as a function of their confidence in their ability to solve a problem correctly. In the present study, children more frequently rearranged bills on the more complex problem, a strategy that, in turn, appeared to reduce the difficulty of this problem for children.

In sum, the present study provides insight into the relation between children's engagement with everyday practices and their developing mathematical understandings. Children generate mathematical problems as they participate in cultural practices such as candy selling. These problems are linked to both larger social processes, such as the inflated monetary system, and to local conventions that arise in the practice, such as the ratio conventions for retail pricing. In addressing these problems, children create new problem-solving procedures and understandings, procedures and understandings well adapted to the exigencies of their everyday lives.

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