CULTURE AND COGNITIVE DEVELOPMENT

Studies in Mathematical Understanding

Geoffrey B. Saxe

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Preface

This volume has taken form through the help and support I received from various teachers, colleagues, friends, and family. I am grateful to them all.

As a graduate student, I studied with Jonas Langer; his rich insights into problems of cognitive development continue to be important to me in working through the conceptual problems that I have struggled with in this volume and in my previous work. I also owe much to other individuals during my undergraduate and graduate work in these earlier years. Elliot Turiel was influential in introducing me to structural-developmental approaches to cognition, and under his supervision while I was an undergraduate, I was introduced to problems of extending structural-developmental approaches to field settings in a project on social cognition in Lower Kalskag, Alaska. His more recent work on domain distinctions in developmental analyses of social cognition has continued to influence my own thinking. Allen Black, and a community of graduate students working with Jonas at Berkeley provided a wonderful intellectual environment to pursue questions of cognitive development, and much of my own ways of conceptualizing problems of cognitive development has its roots in weekly meetings with Jonas and these friends.

I have also benefited by contact with various colleagues in my postdoctoral years. As a postdoctoral fellow under a National Institute of Mental Health training grant, I was introduced to problems of comparative research through the study of atypical cognitive development and the breakdown of cognitive functioning following brain injury at The Children's Hospital Medical Center in Boston under the guidance of Peter Wolff and at the Boston Veteran's Administration Hospital under the guidance of Howard Gardner. In the Papua New Guinea work that I summarize in the volume, I am indebted to the help and

support of David Lancy, Randall Souviney, Tom Moylan, Virginia Guilford, Marshall Lawerence, and numerous other unnamed individuals. During my years on the faculty at the Graduate Center of the City University of New York, short but fruitful discussions with Michael Cole and Joseph Glick were important to me.

In accomplishing the body of the work on child candy sellers described in this volume, I benefited greatly from the supportive research environment created by Analucia Schliemann, Terezinha Carraher, and David Carraher at the Universidade de Pernambuco in Recife. Luciano Meira, a student at the Universidade de Pernambuco, assisted in many phases of the conduct of this project. Though a masters student at the time, Luciano functioned in many ways as an extraordinarily able colleague. In addition, Wilher dos Santos, Anna Ruiz, Danielle, and Marcia were students that assisted in field work and interviews throughout the project, and the high quality of data on which this volume is based collected is due to their fine efforts. Scott Lewis, a graduate student at UCLA, was very helpful in administering the interviews to the U.S. schooled sample described in Part IV.

Maryl Gearhart, Steven Guberman, Joe Becker and Marta Laupa each read and commented on some parts of or various drafts of this manuscript. In addressing their criticisms, I have been led to new insights, and I was able to make this manuscript more readable and coherent.

I am deeply grateful for the patience and support of my family—Maryl, Josh, and Ben (who was born a Brazilian)—on which this volume has taken a toll. Maryl, in particular, has played many roles in my research activities since my early work in Papua New Guinea—wife, friend, close colleague, teacher, and constructive critic. She deserves special thanks for much that is good about this work and my prior work.

Finally, the support that my parents and sister have provided over the years has been instrumental to me in forming and accomplishing goals that have led to this volume. From my first trip around the U.S. as a young teenager to a stay in a remote part of Alaska as an undergraduate to my more recent efforts in non-Western and more Western settings, I have felt warmly supported and encouraged by each of them.

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Spencer Foundation, and the last phases in the writing and preparation of this manuscript were conducted while I was supported by grants from the Spencer Foundation (#M890224) and the National Science Foundation (#MDR–8855643). The opinions expressed in this volume are my own and not necessarily those of the funding agencies.

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CULTURE AND COGNITION: A METHOD OF STUDY

Some years ago, I visited a Papua New Guinean highlands group, the Oksapmin, for the purpose of studying the development of mathematical understandings in a non-Western culture. As a student of cognitive development, I was struck by the differences between the Oksapmins' indigenous mathematical practices and my own. In house building, arrowhead making, string bag weaving, and counting, the Oksapmins' approach to solving mathematical problems of measurement and numeration involved very different ways of thinking and very different procedures for accomplishing everyday problems (Saxe, 1982). For instance, Oksapmin often conceptualize numerical and measurement problems in terms of an indigenous, 27-body-part number system with no base structure. A number is expressed by pointing to a particular body part (like the neck) and saying the body-part name.

I had two initial reactions to the mathematical practices of the Oksapmin which were linked to my graduate training in developmental psychology, training in the structural-developmental tradition of Piaget and focusing on mathematical cognition. The first reaction was an intellectual excitement: The same mathematical operations of correspondence and measurement captured in Piagetian psychogenetic analyses that I had studied in Western children were apparent in the activities of a people from an extraordinarily foreign culture. The second was an intellectual frustration: The aspects of cognition and the texture of everyday life in Oksapmin that seemed so marvelously different from that of the West—like

the Oksapmins' use of their numeration system in everyday practices—were not captured by Piagetian core constructs like conservation. While Piaget's epigenetic constructivism—the thesis that the individual generates novel intellectual structures by reorganizing prior knowledge to resolve contradictions—appeared critical to me for conceptualizing developing cognitive processes, reducing thought to a small set of Piagetian categories seemed weak as a method for the study of culture-cognition relations.

It has been 10 years since my first visit to Oksapmin. The volume that follows illustrates my efforts to synthesize a research program that provides insight into the distinctiveness of children's cognitive development across cultures but at the same time reveals universal regulative processes which transcend cultural boundaries.

In the first part of this volume, I introduce a general analytic model that targets cultural practices as important contexts for study. In subsequent parts, I apply the model to a single cultural practice—candy selling—as it has emerged in the lives of children living in northeastern Brazil. In candy selling, the relations between culture and cognitive development stand out in particularly clear relief and are particularly amenable to study.

1 Culture and Cognitive Development

Treatments of cognitive development can be understood as rooted in one of three fundamental views on the character of knowledge. Each view carries with it both problems and advantages for an adequate account.

The empiricist view is that the environment is the source of knowledge, and through experience children's knowledge comes to reflect the environment with increasing exactness. The position has its roots in philosophy (e.g., Locke) and is also formulated in modern treatments of learning (Bijou & Baer, 1961; Gagné, 1985). The promise of these treatments is to explain the way children's knowledge is shaped and organized directly by experience. But to date, the promise is far from being realized and many have argued that, in principle, rational thought structures like logical deduction cannot be accounted for solely by reference to the environment (e.g., Chomsky, 1957; Piaget, 1970).

The nativist position acknowledges the need for fundamental knowledge structures to organize experience into categories and logical systems, and its claim is that these structures are a hereditary endowment. Again, the position has ancient philosophical roots (Plato) and finds its current articulation in treatments of language (Chomsky, 1972) and cognition (Fodor, 1983). While such models explain the independence of rational cognitive structures and experience, they at the same time do not offer compelling treatments of development nor the variability of cognitive forms across cultures.

The constructivist position—in which the treatment I develop in this volume has its roots—is that fundamental aspects of knowledge neither come preformed in the genes nor in the environment, but are actively constructed by the developing individual. The aim of constructivist accounts is to show how novel cognitive structures emerge as a function of the developing subjects' commerce with a

social and physical environment, and the focus is on explaining cognitive development with reference to principles of self-regulated change and interaction.

The problem which is the focus of the volume is to understand the interplay between sociohistorical and cognitive developmental processes. Most fundamentally, my concern is to understand how artifacts and forms of social organization—products which have emerged over the course of social history—come to be interwoven with and are intrinsically related to the nature of children's intellectual constructions. These historical products may be conceptual as in the case of scientific concepts entailed in Newtonian mechanics, symbolic forms as in the case of numeration or writing systems, or material as in the case of tools like the lever or the computer.

In the following discussion, I consider the problems and prospects in the ways researchers in the constructivist tradition have conceptualized and studied the interplay between sociohistorical processes and cognitive developmental ones. While constructivist treatments share core assumptions, they often differ in their analytic units and in their levels of analysis. All have had difficulty in producing rich and systematic conceptual frameworks for the analysis of intrinsic relations between cognitive developmental and sociohistorical processes. After reviewing key features of these treatments in this chapter, I then turn, in Chapter 2, to my effort to work toward a more complete framework for the analysis of the interplay between cognitive developmental and sociohistorical processes.

CONSTRUCTIVIST APPROACHES

A basic assumption of constructivist treatments is that individuals create new knowledge in their goal-directed activities; in turn, new knowledge leads subjects to identify new goals. Cassirer (1957), in his philosophical treatment, expresses the constructivist dialectic between conceptual advances and goal-directed activities extraordinarily well.

> [E]ach newly acquired concept is an attempt, a beginning, a problem; its value lies not in its copying of definite objects, but in its opening up of new logical perspectives . . . one of its essential tasks is not to let the problems of knowledge come prematurely to rest, but to keep them in a steady flux, by guiding them toward new goals. . . . Here again we find that the concept is far less abstractive than prospective; it not only fixes what is already known, establishing its general outlines, but also maintains a persistent outlook for new and unknown connections. (Cassirer, 1957, p. 306)

Thus, from the constructivist perspective, goals themselves are rooted in individuals' understandings.

Empirical research on culture and cognitive development has been influenced by two constructivist treatments which differ in their description of the role of

social processes in the development of the individual's self-regulative autonomous reasoning. One treatment is associated with the structural developmental approach of Piaget, and the other is associated with the sociohistorical approach of Vygotsky.

Piaget and the Structural-developmental Approach

Piaget's treatment of cognitive development is rooted in a neo-Kantian epistemology in which a principal assumption is that the world is not known directly but is assimilated by intellectual structures. Kant and Piaget shared a concern to understand how the subject comes to know the necessity of propositions about logico-mathematical and physical phenomena. The solution for both was in the properties of these cognitive structures. Piaget, however, did not share Kant's monism and took a developmental perspective. Using empirical research, he produced evidence supporting a sequence of four stages of cognitive development that extended from infancy through adolescence.

In Piaget's analysis, each successive stage constituted a new cognitive equilibrium, and for each stage Piaget's central analytic concern was to present an analysis of its structural or formal properties. In Piaget's scheme, reality for the infant at birth is no more than the extension of its hereditary reflexes, like sucking and grasping (Piaget, 1954, 1963). In the course of progressively more complex interactions with its environment, the infant transforms these hereditary reflexes into cognitive structures which make it possible, at about 18 months, for the infant to "re-present" experience, and semiotic systems (imitation, imagery, language) begin to emerge (Piaget, 1962). The representational capacity carries with it, however, new problems of coordinating representations, problems which are not solved until the next stage when concrete operational classificatory and relational structures emerge in middle childhood (Inhelder & Piaget, 1969; Piaget, 1952). Concrete operational structures provide the basis for a wide range of novel and stable concepts (e.g., quantity conservations, Euclidean and projective spatial understandings). It is not until early adolescence, however, that the individual constructs an integration of concrete operational class and relational operations into a system of formal operations which is the basis for hypothetico-deductive reasoning (Inhelder & Piaget, 1958).

To explain progress through his stages, Piaget, like Cassirer, argued that the subject is purposeful, constructing goals based on prior knowledge and creating coherent solutions to these goals in the form of novel cognitive developments. In Piaget's analysis, development proceeds by a self-regulated construction, or "equilibration," a dialectical process in which the subject resolves perturbations in the coherence of his or her structuring activities by coordinating and constructing new, more adequate cognitive structures. For Piaget, it is equilibration that guides the direction and organization of cognitive development (Piaget, 1977).

In Piaget's treatment of equilibration, the interplay between social life and

cognitive developmental processes was not a core concern. Indeed for Piaget, the focus was on the formal properties of action without regard for the situatedness of actions in a sociohistorically articulated web of meanings. Nonetheless, questions of social influences on cognitive development have emerged in the work of various researchers who have attempted to extend Piaget's analyses to social processes, and Piaget himself has noted the effect of social processes on the rate of progress through his stages (Piaget, 1966, 1972). However, we find in these extensions that social life is related to cognitive development as an external process, and the way sociocultural life may be deeply interwoven with the character of intellectual functioning is unanalyzed.

Empirical Research on Culture and Cognitive Development Related to the Piagetian Framework

To study sociocultural influences on cognitive development from within the Piagetian framework, researchers have contrasted individuals' performances from different cultural groups on Piagetian tasks (for a review, see Dasen, 1972; Dasen & Heron, 1981). Typically, researchers have focused on the transition to concrete operations, using Piaget's original tasks or slight adaptations of these in varying content areas (e.g., conservation (Laurendeau–Bendavid, 1977; Opper, 1977), classification (de Lacey, 1970), motion (Za'rour & Khuri, 1977), space (de Lemos, 1974). This literature has revealed both cultural similarities and differences. Age norms from many groups suggest that children progress up through Piaget's stage of concrete operations, though there is little documentation of the emergence of formal operational structures across groups (see Neimark, 1975; Piaget, 1972). Further, the age norms for passing concrete operational tasks vary widely across cultures. Whether these cross-cultural differences reflect merely lack of appropriate accommodations of method to the different cultural contexts or actual differences in conceptual development is not entirely clear and has been the subject of considerable discussion (see Hallpike, 1979; Jahoda, 1980; Kamara & Easley, 1977; Nyiti, 1982; Piaget, 1972). Regardless, this literature sets the stage for more focused investigations of the ways that dimensions of sociocultural life may be associated with progress through Piaget's stage sequence. I will briefly consider three such dimensions.

Cultural Practices. One sociocultural dimension isolated for study is participation in cultural practices thought to favor equilibrations. Researchers have analyzed relations between participation in such practices as pottery making and children's understanding of mass conservation in rural Mexican groups (Price–Williams, Gordon, & Ramirez, 1967; Steinberg & Dunn, 1976), practices of economic exchange and children's understanding of number conservation in West African groups (Posner & Baroody, 1979), practices of hunting and nomadic life style on spatial concepts in Canadian Eskimos (as opposed to

agricultural and sedentary groups; Dasen, 1975), and schooling and various concrete operational concepts (Goodnow & Bethon, 1966; Laurendeau–Bendavid, 1977; Mermelstein & Shulman, 1967). The results of studies on practice participation and schooling are varied. Some indicate effects on concrete operational concepts; others do not. Rarely is there a detailed analysis of which aspects of practice-linked experience that were related to cognitive developmental change, and herein lies a problem with this focus. Practices are "packaged variables" (Whiting, 1976), and efforts to unpack a practice requires an analytic model that links the structure of practices with the structure of cognitive developmental processes, an analytic model that has yet to emerge in any sophisticated form within the Piagetian framework.

Social Interactions. A second sociocultural dimension isolated for analysis is social interaction (Doise & Mugny, 1984). Unlike the practice-based research, the social interaction studies are typically laboratory-based. Investigators typically draw activities from Piagetian assessment tasks and instruct dyads to work collaboratively to reach agreement on a solution. Such studies often make use of an experimental design in which children are randomly assigned to experimental (social interaction) and comparison (no social interaction) groups, and interactions in the experimental groups are often videotaped for later analysis. These studies sacrifice the naturalistic features of the cultural practice research. However, they typically provide a more detailed account of social interactional processes hypothesized to be implicated in stage change, most notably social conflict which, it is argued, sets in motion the process of equilibration. Using this kind of paradigm, researchers have studied spatial perspective taking (Bearison, Magzamen, & Filardo, 1986; Doise & Mugny, 1984; Doise, Mugny, & Perret–Clermont, 1975, 1976), proportionality in the balance (Damon, 1988), conservation (Silverman & Geiringer, 1973; Silverman & Stone, 1972), and classification (Valiant, Glachan, & Emler, 1982).

The findings from Piagetian social interaction research have often revealed effects for collaborative interactions, particularly interactions in which a modicum of social conflict is generated during joint problem solving (Bearison et al., 1986). But what we may lose access to in laboratory situations is the sociohistorical context of the interactions, a context in which elaborated conventions and activity structures give meaning to and modify the organization of interactions.

Sign-using Activities. A third sociocultural dimension isolated for study is the acquisition—or lack thereof—of particular sign forms like language or numeration and the influence of these sign forms on the emergence of cognitive structures. For Piaget, cognitive structures are not constructed through the acquisition of sign forms like language and numeration, but through the equilibration process. Thus, from the Piagetian point of view, progress through his stage

sequence could not have its origins in, nor be dependent on, the acquisition or particular sign forms. To evaluate Piaget's claims, some researchers have investigated the impact of the acquisition of linguistic and numerical forms on concrete operational concepts.

In a well-known training study focusing on the acquisition of comparative terms, Sinclair (1967) taught children the meanings of such comparatives as "more" and "less" and then assessed children's understanding of Piagetian quantity concepts like conservation. Consistent with Piaget's claim about sign forms and stage progression, Sinclair found that the training had only limited effects: Only children at a transitional stage showed signs of shifting to conservation understandings. Additional language training studies in which the focus has been on training linguistic rules for conservation (rather than lexical items) have shown somewhat greater success in facilitating the development of conservation performance (see Beilin, 1976, for a review).

Comparative research has also been used to investigate sign form–cognition relations from the Piagetian perspective. Furth (1966) found that deaf children, despite their failure to develop spoken language, developed Piagetian concrete operational concepts, though at later ages than hearing children. Lancy (1983), in a study of Papua New Guinean cultural groups that used different kinds of numeration systems—from base-10 verbal systems to body systems with and without base structures—found no influence of numeration system on the ability to pass Piagetian concrete operational tasks.

As a whole, these studies indicate that while symbolic forms may affect the rate of development, they are consistent with Piaget's writings on the primacy of the endogenous process of equilibration.

Piagetian Theory and Sociocultural Processes

The Piagetian based cross-cultural research clearly lends support to the constructivist thesis of universal self-regulated processes. This body of research, however, has not led, in any rich sense, to advancing our understanding of the interplay between sociohistorical processes and cognitive developmental ones. The crux of the problem is that by reducing cognitive development to descriptions of abstract cognitive structures, we gear analytic categories to aspects of cognitive functioning that do not reflect the socially textured goals of everyday life and associated cognitive constructions.

This critique of Piagetian research on sociocultural processes is in fact applicable to a wide range of research on cognitive development. Across most research efforts, to the extent that sociocultural processes are addressed, the analytic approach is to dissect both cultural and cognitive phenomena into separate sets of elements in which the social properties of cognition are no longer recoverable. Thus, for example, within current information-processing perspectives, such elements may include speed of cognitive processing (Kail, 1986) or

amount of working memory (Klahr, 1980); within the cognitive style perspective, elements consist of constructs of field dependence or field independence (Witkin & Berry, 1975; Witkin, Dyk, Faterson, Goodenough, & Karp, 1962); and within the Piagetian tradition, elements may include any of the Piagetian concrete operational concepts (Dasen, 1972; Opper, 1977). While there may be attempts to identify cultural influences on independently defined cognitions through the external connections of correlational analyses as we have seen in much of the Piagetian work, these efforts cannot lead to analyses of the intrinsic relations whereby cultural and cognitive developmental phenomena are constitutive of one another.

In turning now to Vygotsky's constructivism and related work, we find an analytic tack that differs critically in the way social processes are conceptualized in relation to cognitive developmental ones. In the Vygotskian approach, we see an analytic framework elaborated that has as its central focus intrinsic relations between cognitive developmental processes and sociohistorical ones.

Vygotsky

Vygotsky, whose psychological writings have epistemological underpinnings in the works of Marx and Hegel, outlined a specifically sociohistorical approach to cognitive development (Vygotsky, 1978, 1986). For Vygotsky, a central problem was to understand how infants who are initially impulsive, responding directly to environmental stimulation, gain control over interactions with their environments. The solution was to be found in children's appropriation of sociocultural artifacts and supports—sign forms, assistance from others, scientific concepts—to mediate their interactions with the environment. Vygotsky's view was that the appropriation of cultural artifacts liberates children from direct stimulus control and also creates an intrinsic link between cognitive development and culture.

Sign Forms

For Vygotsky, the emergence of spoken language in the children—one of various sign using activities—is at the heart of the interplay between natural and sociohistorical processes of the individual's development. Early in development, Vygotsky argued, thought and speech have separate roots: There is a preintellectual phase in speech development and a preverbal phase in the development of thought. During early childhood, these processes begin to penetrate one another; the child uses telegraphic speech first to accompany problem solving, and gradually uses speech to help solve and then to plan solutions to problems. Eventually, we see the telegraphic or "egocentric" speech of toddlerhood gradually go "underground," transforming into covert "inner speech." Thus, what was once a social artifact external to the child, is gradually transformed by the child, first into an external aid which helps organize problem solving and later into a core ingredient of conscious thought.

For Vygotsky, speech was one of a variety of cultural sign forms that displayed a complex transformation on its inward trajectory. Vygotsky's analysis of sign form use in the emergence of mediated or "voluntary" memory presents an interesting parallel. In one study, Vygotsky's purpose was to show that, as in the case of speech, the functional relation of the sign form to basic intellectual processes shifts with the inward movement. Sign forms begin as merely juxtaposed with goal-directed efforts; they are subsequently organized consciously by the child into a part of goal-directed activities as they become external solution means; finally, they become transformed into automatic and abbreviated covert mental activities. To illustrate this developmental phenomenon in the case of memory, Vygotsky presented individual children with a game-like laboratory task: Children were required to answer questions posed by the interviewer. Many of the questions required children to respond with color names, some of which they were forbidden to say. In one condition, children were also presented with a set of auxiliary materials—color chips—that could be used as a sign form. Children in the youngest group often found the chips a source of added difficulty and did not attempt to use them as a means of task solution; these children committed just as many errors on the chip and no-chip conditions. Children in the middle age groups performed better in the chip condition than in the no-chip condition; they constructed strategies to use the chips as a sign form to aid their memory. In the oldest age groups, performance was very good across chip and no-chip conditions. Vygotsky argued that adults had generated internal strategies akin to external chip manipulation to solve the task.

Social Interaction

For Vygotsky, social interactions were a critical vehicle whereby natural processes in cognitive development were redirected by social and historical influences. In social interactions, Vygotsky argued, "zones of proximal developments" are created. A zone of proximal development was defined as the difference between what a child could accomplish unassisted in problem solving and what he or she could accomplish with assistance. Vygotsky argued that socially supported activity in the zone of proximal development awakened and provided paths for intellectual development.

During the past decade, various investigators have conducted studies of adult-child interaction concerned with Vygotsky's notion of the zone of proximal development (Rogoff, 1986, 1990; Rogoff, Ellis, & Gardner, 1984; Rogoff & Gardner, 1984; Saxe, Gearhart, & Guberman, 1984; Wertsch, 1979; Wertsch, McNamee, McLane, & Budwig, 1980) or related constructs (Wood, Bruner, & Ross, 1976; Wood & Middleton, 1975; Wood, Wood, & Middleton, 1978). These studies often reveal that it is quite natural for adults to enter into children's problem solving in ways that enable children to participate with goals and means of achieving them that are more sophisticated than the children establish on their

own. In one example of this work, colleagues and I (Saxe, Guberman, & Gearhart, 1987) studied 2½- and 4½-year-olds and their mothers from working and middle class families. We analyzed the children solving numerical problems unassisted and then we observed them solving the same problems with their mothers' assistance. Our analyses were consistent with Vygotsky's writings on the zone of proximal development. In social interactions with their mothers, children achieved more sophisticated goals than they did on their own. Further, an analysis of videotapes of these interactions revealed a flexibility in the ways in which the goal structure of the task emerged over the interactions. When children were having difficulties, mothers tended to simplify the numerical goals of the task, and when children were doing well, mothers tended to focus on more complex numerical goals. Such adjustments were largely independent of both age and social class. Thus, in adult-child interactions, children are accomplishing goals that are at once linked to their own constructive efforts and to sociocultural life.

Spontaneous and Scientific Concepts

Scientific concepts are interconnected and comprehensive systems of understandings which have been elaborated and refined over the course of social history. The conceptual systems entailed in Newton's mechanics or Marx's *Das Capital* are examples. Just as the child's appropriation of sign forms and assistance from others provide critical avenues for developmental processes, so too do children's learning of scientific concepts. Like sign forms and assistance from others, scientific concepts are not simply internalized, but undergo a complex transformation in their inward movement from artifacts external to the child's activity to mental processes which are interwoven with the child's intellectual functions.

Vygotsky argued that to understand the acquisition of scientific concepts requires a perspective which also includes an analysis of children's spontaneous concepts. For Vygotsky, children construct "spontaneous concepts" from the bottom–up: Children's everyday experiences are inherently local, occurring in particular contexts, and the concepts which children thus form are ones linked to situations—the bike stops more quickly when the hand brake is squeezed harder; the soccer ball goes further when it is kicked harder. Such spontaneous concepts are rich in meaning for children, but they are local and not linked with one another in general systems of interrelated understandings. Children only gradually draw connections between such isolated experiences in forming more general concepts. For Vygotsky, children construct "scientific concepts" from the top–down. In school, children are presented with scientific concepts, like those entailed in Newton's mechanics. Initially, such concepts are abstract but empty; while the child may learn the "syntax" of the relations of a network of concepts, these concepts have little apparent relation to the child's spontaneous concepts.

The development of scientific concepts proceeds downward toward the application to the situated phenomena of handbrakes and soccer balls.

For Vygotsky, it is in the interaction between the top–down movement of scientific concepts and the bottom–up movement of spontaneous concepts that we find intrinsic links between the individual and social history. In their interaction, spontaneous concepts enrich scientific concepts with meaning and scientific concepts offer generality to the development of spontaneous concepts—again, a melding of the individual with the sociohistorical.

Vygotskian Insights Applied to Cultural Practices

Though Vygotsky did not consider directly cultural practices in his writings, researchers influenced by Vygotsky have elevated practices to an important unit for analyzing culture-cognition relations. Researchers have targeted for analysis the mathematical problem solving of Kpelle rice farmers of central Liberia (Cole, Gay, Glick, & Sharp, 1971; Gay & Cole, 1965), Vai tailors' strategies for measurement (Lave, 1977), Southern California housewives' strategies for finding best buys in the supermarket (Lave, 1988), dairy workers' strategies for loading crates (Scribner, 1986), horse racing strategies for betting (Ceci & Liker, 1986), Weight Watchers dieters' mathematical strategies for solving measurement problems (de la Rocha, 1983), and schoolchildren's arithmetic from traditional societies (Brenner, 1985). In comparison with the Piagetian practice-based studies noted earlier, the Vygotskian studies often provide a more detailed analysis of the kinds of cognitive forms individuals structure to accomplish cognitive functions linked to practice participation, and the descriptions produced help us understand interdependencies between particular sociocultural and cognitive developmental processes. For instance, whether we read Scribner's descriptions of dairy workers' strategies of the use of the particular arrangement of a crate to organize a solution to a problem of retrieving a quantity of milk, or Lave's description of tailors as they make use of unique mathematical procedures to ply their trade, we find in these studies analyses of the varied mathematical forms with which adults address problems linked to their everyday activities. Nevertheless, these studies fall considerably short of addressing the core problems elaborated in the analytic writings of Vygotsky and Piaget and central to the present work.

Most centrally, these studies do not treat cognition from a developmental perspective, a perspective in which cognitive forms are understood as evolving in a complex psychogenetic process, shifting in function over the course of their evolution. For instance, we rarely observe individuals sampled at different ages or at different points in their acquisition of a trade. Even when such a sampling procedure is used, we do not find analysis of the shifting character of cognitive forms as these forms are interwoven with shifts in people's goal-directed activities (Ceci & Liker, 1986; Lave, 1977; Luria, 1976).

Further, because development is not a principal focus in the practice-based studies, we see little analysis of enculturative processes whereby children's understandings become interwoven with the cognitive achievements and practices of their social group (see Greenfield & Childs, 1977, for a noteworthy exception to this critique). Consider the situation of the child novice entering the practice of potting, weaving, or street selling. The child has little knowledge of how to participate in the practice and has not structured the sometimes sophisticated cognitive forms that are required to solve conceptual problems that emerge in the practice. In order for the child to begin to participate, there must be routes that ease the conceptual burdens. For instance, other more experienced participants may modulate the conceptual problems which novices address, and their modulation may be akin to that detailed in research on the zone of proximal development that has been reviewed. An analysis of such *in situ* processes would be critical both to a treatment of the actual goals that children of different ages or levels of expertise are addressing in a cultural practice and to understanding how cognitive forms of the individual become interwoven with those of the larger social group.

Finally, because children are not a principal focus for analysis in the practice-linked work, we find no analyses of the interplay between scientific and spontaneous cognitive forms. Perhaps the most significant question here is the way in which school learning interacts with the kinds of understandings children generate through their participation in everyday cultural practices. Despite the importance of this question from both the point of view of education and treatments of sociocultural processes in cognitive development, we have little empirical research in this area.

Cultural Practices: Toward a Developmental Perspective

In a seminal volume, *The Mind of Primitive man,* Franz Boas (1911) pointed to the critical status of cultural practices in treatments of culture and cognitive development, arguing that people generate intellectual skills in the context of the practices with which they are engaged. In his observations of native Americans, he pointed out that cultural practices vary in complexity both within and across native American groups, and the character of people's intellectual adaptations vary accordingly. We find support for Boas's conclusion in the Piagetian and Vygotsky-based research on cultural practices, though we are some distance from understanding the dynamics of the interaction that leads to these associations.

In the next chapter, I outline a research framework for gaining insight into the interplay between sociocultural and cognitive developmental processes through the analysis of practice participation. The framework shares the underlying constructivist assumptions of the Piagetian and Vygotskian formulations, and, with respect to core constructivist assumptions, the model presented here is consistent with both approaches. However, the framework presented targets a level of

analysis that is not addressed by either of these formulations. Unlike the Piagetian approach, my concern is to treat cognitive development on a level of analysis in which activity-in-sociocultural context is a critical focus and cognitive developmental processes are analyzed with reference to these contexted activities. Unlike the Vygotskian writings, which do not develop core developmental and sociocultural theoretical constructs with reference to systematic analysis of core domains of knowledge, the present approach is concerned with a systematic analysis of mathematical cognition that integrates cognitive developmental and sociohistorical perspectives.

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