Culture, Language, and Number

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Some time ago, Bryant (1997) remarked, "Piaget and Vygotsky set the scene for much of the work that has been done over the last twenty years or so on children's mathematical understanding. (p. 142)". Today, Piaget and Vygotsky's conceptual and empirical frameworks still define principal contours of contemporary work on cognitive development. In introducing this section on culture, language, and number with chapters by Okamoto, Towse, Nunez & Marghetis, and Sturman, I situate the authors' contributions in relation to Vygotsky's and Piaget's seminal writings and some contemporary strands of empirical and conceptual inquiry.

Vygotsky and Piaget: Seminal Formulations

In their treatments, Piaget and Vygotsky each addressed questions about the origins and development of cognition. Although each used a developmental method, their principal foci differed. Vygotsky's treatment is geared for understanding the situatedness of cognitive development in socio-historical processes; Piaget's contributions focused largely on the individuals' constructions of cognitive structures across varied knowledge domains, including number. Themes developed in Vygotsky's and Piaget's writings are reflected across most of the chapters in this section, and in each chapter, the Piagetian and Vygotskian themes appear interwoven with one another in interesting ways.

Vygotsky's Cultural-Historical Perspective

As a part of his general treatment of cognition, Vygotsky distinguished between "natural" and "cultural" lines in children's cognitive development (Vygotsky, 1986). For Vygotsky, the *natural line* began with native capacities that include involuntary memory and a host of reflexive processes that organized the infants' world, whether visual, tactile, or auditory. By their nature, these cognitive and perceptual processes were not under conscious control; rather they were elicited by environmental stimuli or generated by impulse.

The *cultural line* of development is captured in Vygotsky's treatment of mediation. For human cognition to emerge, young children had to break the link between the stimulus world and immediate responding. Vygotsky argued that children began to interrupt the stimulus-response link by consciously drawing upon aspects of their environments (social, physical, linguistic) to mediate their interaction with their immediate worlds. In the early mediational act, the child begins to take control and organize their own responses via historically elaborated representational and knowledge systems. An example is children's early use of speech to talk through a problem or to plan.

In Vygotsky's treatment, the child's early use of mediational forms was the beginning of a long and complex developmental process in the "cultural line" of development. In the case of language, for example, Vygotsky argued that early speech initially served to support social contact and gradually became incorporated as speech used to describe, and then to organize their activity. This "egocentric speech" used to organize activity undergoes further transformations, becoming deeply interwoven with properties of thought and action, developing into "inner speech." Although Vygotsky contributed little to empirical research on children's developing understandings of number, in the chapters to follow, investigators bring forward an interest in mediation and number. Foci include linguistic group's number word systems (Okamoto, Towse) and conceptual metaphors that serve as a basis to make sense of and make inferences about a numerical world (Nunez & Marghetis).

Piaget's Treatment of Number Development

Piaget's treatment for most of his long career was focused on qualitative shifts in cognition across varied domains of knowledge. One central concern was with children's numerical understandings. Piaget's principal monograph on number sought to illuminate origins of ideas related to numerical unit and one-to-one correspondence (Piaget, 1952). Piaget documented both continuities and discontinuities in the transformation of these fundamental numerical ideas as well as articulated a dialectical treatment of developmental change.

To illustrate areas of complementarity in Piaget's and Vygotsky's writings, which are unified in several of the chapters to follow, consider an anecdote that Piaget presented in a published lecture (Piaget, 1970). Though the anecdote and its interpretation is Piaget's, the anecdote is also interpretable by Vygotsky's framework. In the anecdote, Piaget conveys an account told to him by a mathematician friend.

> When he was a small child, he <the mathematician> was counting pebbles one day; he lined them up in a row, counted them from left to right, and got ten. Then, just for fun, he counted them from right to left to see what number he would get, and was astonished that he got ten again. He put the pebbles in a circle and counted them, and once again there were ten. He went around the circle in the other way and got ten again. And no matter how he put the pebbles down, when he counted them, the number came to ten. He discovered here what is known in mathematics as commutativity, that is, the sum is independent of the order. But how did he discover this? Is this commutativity a property of the pebbles? It is true that the pebbles, as it

> were, let him arrange them in various ways; he could not have done the same thing with drops of water. So in this sense there was a physical aspect to his knowledge. But the order was not in the pebbles; it was he, the subject, who put the pebbles in a line and then in a circle. Moreover, the sum was not in the pebbles themselves; it was he who united them. The knowledge that this future mathematician discovered that day was drawn, then, not from the physical properties of the pebbles, but from the actions that he carried out on the pebbles. (Piaget, 1970)

Piaget's use of the narrative illustrates his constructive perspective about the origins of number. For Piaget, fundamental ideas like commutativity are not native, nor are they contained in a linguistic system like number words, for it was the child who generated the idea through his own actions. The narrative clearly conveys Piaget's thesis that number is not "in the environment" or "in the head" but emerges in an interaction between properties of the world and the actions of the subject

At the same time, the anecdote also points to the utility of Vygotsky's treatment of a cultural line of development. After all, the boy is a participant in a cultural world in which a number word system has its roots, a number word system that both enables and constrains his activity. Indeed, number word systems are not "immaculately conceived" but rather have developed over particular cultural histories (Saxe & Esmonde, 2005; Saxe & Posner, 1982). Further, not only are number systems historically situated, but also the very numerical problems with which children engage are linked to cultural and social processes (Saxe, 2012).

Some Contemporary Strands of Research that Engage Culture, Language, and the Development of Numerical Cognition

In the chapters that follow, the reader will find that the authors extend the themes of mediation (akin to Vygotsky) and developing structures of numerical thought (akin to Piaget). At the same time, the authors also contribute to contemporary strands of research on relations between culture, language, and the development of numerical cognition. By way of introduction to the chapters, I provide a brief overview of these areas of contemporary work.

Research on Language and Number Development

A first strand of contemporary work focuses explicitly on the way the number word registers of linguistic groups may mediate the development of numerical thinking, a broad area of theorizing and empirical research (cf. Bowerman, 1996; Everett, 2005; Frank, Everett, Fedorenko, & Gibson, 2008; Lancy, 1983; Whorf, 1956). Some languages, like many of the Papua New Guinea highlands, for example, support body part counting systems (cf. Laycock, 1975; Saxe, 2012). Other linguistic groups use linguistic registers, but these groups vary in the structure of the lexicon (e.g., whether they use a base or multi-unit structure) as well as the magnitude that can be counted with a system (Dehaene, Izard, Spelke, & Pica, 2008; Everett, 2005; Saxe &

Posner, 1982). A general issue in this literature is whether and/or in what way the number system children acquire influences the character of their numerical thought.

In a series of studies to which a number of chapters that follow make reference, Miura and her associates (Miura, 1987; Miura, Kim, Chang, & Okamoto, 1988; Miura & Okamoto, 2003; Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999) investigated the role of number word systems on children's developing conceptual understanding of number representations. Miura's focus picks up Vygotsky's treatment of relations between language and thought; at the same time, it extends Piaget's treatment of numerical unit to a children's developing conceptual differentiations and coordinations of units and multiunits.

Miura began by noting that languages that are derived from Ancient Chinese (like Japanese, Chinese, and Korean) have a numerical register organized by base-10 that is entirely regular and maps directly to the Hindu-Arabic written base-10 system. Other non-Asian systems, like English also make use of a base-10 register but with many irregularities that make the mapping to the written system less transparent. In a series of studies, Miura asked, does the number word system that children are acquiring influence their developing conceptualization of number? To address the question, Miura created tasks in which children in the early elementary grades from different language backgrounds would be required to show an interviewer different numbers using base-10 blocks, unit blocks (cubes) and 10-blocks (ten times longer than the unit blocks). Miura and her associates' principal finding was that children speaking languages rooted in Ancient Chinese much more frequently use multi-unit representations to show two-digit numbers, whereas the children from comparison language groups tended to produce non-multiunit representations, simply counting unit blocks.

Subsequent work has built upon or questioned Miura's interpretation of her findings about the role of a number word system in conceptual understanding; some of the authors of this work are also are contributors to chapters in this section. Among the issues that are raised include: how robust is the effect that Miura reports? Can task variation or particular treatments reduce the effect easily? If the effect is robust, how general is it? Is the effect limited, for example to performance on Miura's type of task? Does it extend to place value understandings? Does it affect a range of conceptual developments in domain of mathematics? Finally, does the effect of language account in any way for the results of international assessments of mathematics achievement (TIMMS, PISA), which document higher levels of achievement among students from Asian countries that students from non-Asian countries?

Research on the "Mental Number Line"

A second strand of research focuses on what some have referred to as the "mental number line." Dehaene (1997) posited that people naturally represent numbers spatially on a linear dimension, with the spacing of larger numbers compressed. In a subsequent study, Siegler (2003) generated evidence for a developmental process, beginning with a logarithmic models and shifting to a linear ("accumulator") model, in which numbers progress as a linear function (Gibbon & Church, 1981). In one of Seigler's tasks (2003), participants were presented with a number line marked with a 0 at one end and 100 at the other, and asked their participants to estimate the positions of numbers presented to them. When he tested 2^{nd}, 4^{th}, 6^{th} graders and adults, he found a shifting prominence of how numbers were positioned on the line. For younger populations, numbers were compressed at the higher ends, resulting in a function that appeared logarithmic (like original Dehaene et al findings). For the older participants, individuals produced linear functions in spacing numbers.

In some important respects, the literature on the mental number line echoes themes in Vygotsky's and Piaget's developmental approaches. On the one hand, Dehaene argues that the logarithmically scaled number line is a "natural" or "intuitive" native cognitive capacity with which we organize the quantitative world, much akin to the way Vygotsky discusses native capacities and a natural line of development. Further, it may be, following Vygotsky in part, that the shift to a line that is organized by a linear function may be understood as a shift in mediational processes. Consider, for example that Dehaene and his associates investigated the use of modified number estimation number line tasks with a remote Amazonian group. The investigators found that adults in the Amazonian group perform like children in the US, their estimates conforming to a logarithmic model. Dehaene concludes from this that the logarithmic function is the intuitive one and perhaps universal (Dehaene, et al., 2008). One might imagine that the mediational shift is linked to participation in schooling. From Vygotsky's perspective, one might conjecture that children may be drawing upon artifacts like rulers to mediate their estimates of positions in a one-dimensional space.

The shift from logarithmic to linear functions is also, in some respects, in accord with Piaget's early findings on the development of spatial cognition. Early in development, children's spatial constructions have properties of a topological space, with order and continuity respected, but not measured distances. With age, Piaget has produced a great deal of data to support the argument that children construct a Euclidean space, with dimensional frames of reference and metric properties (Piaget & Inhelder, 1956).

Assessment

Assessing cognition is a dominant concern in cultural and linguistic approaches to numerical cognition, and much has been written about the topic (a useful reference on this is Ginsburg, 1997). For their part, Vygotsky and Piaget each elaborated a variety of assessment methods keyed to their conceptual frameworks.

To explore the shifting properties of mediational activity, Vygotsky (1978, 1986) used a technique that he referred to as the "method of double stimulation." With this technique, the child was presented a primary problem (primary stimulus) and artifacts that could be drawn into and mediate the solution in order to restructure the problem to achieve an adequate solution. An example is Vygotsky's study of mediated memory – children of different ages were engaged with a game in which they could not say particular colors and could not say any color twice. The interviewer would proceed to ask questions like, "What color is the sky?" and "What color are your shoes?" To support their efforts, children were presented, as auxiliary stimuli, color chips that could be drawn into their efforts to support their play. What Vygotsky found in this study, as in others, was that with age, participants shifted in the way that they used the chips to mediate their memory for what colors they could or couldn't say. Younger children typically ignored or were confused by the chips; older children used the chips productively; and the still older children and adult group did not rely on the physical chips to mediate their solutions.

Piaget also developed a wide range of methodological techniques. One for which he is particularly well known was the clinical interview (Piaget, 1979). The clinical interview is used to explore the child's thinking much as an anthropologist might explore the phenomenological worlds of people from a different and remote culture. In one approach to the interview, the child is presented with a cognitive tsk and the interviewer explores the approach that the child takes to the task, probing their thinking in ways that maintain rapport and at the same time illuminate the way the task is conceptualized and solved by the interviewee. The interviewer uses non-leading questions and probe quests that help to disambiguate different interpretations of the way the child is conceptualizing and solving a task.

The chapters to follow each present perspectives and approaches to assessment keyed to understand issues of mediation and children's numerical thinking and achievement. Some are "high stakes" assessments used to evaluate national differences in students achievements in the TIMMS and PISA studies. Others are modification of Miura's tasks and study designs; still others involve new designs and approaches to assessing participants' positional estimates for numbers on a number line. Across studies, there are some important methodological issues. Consider one: Claims about a “mental number line” stem from assessment tasks in which children are presented with a linear representation and asked to place numbers on it. Are claims about a “mental number line” warranted from such findings? Do such findings from linear tasks warrant claims about processes native to the central nervous system? Alternatively, is this “mental number line” an artifact that results from presenting participants with a linear display and asking them to produce numerical estimates? In the latter case, there is no “mental number line,” merely an approach to solving a particularly kind of task requiring linear representations.

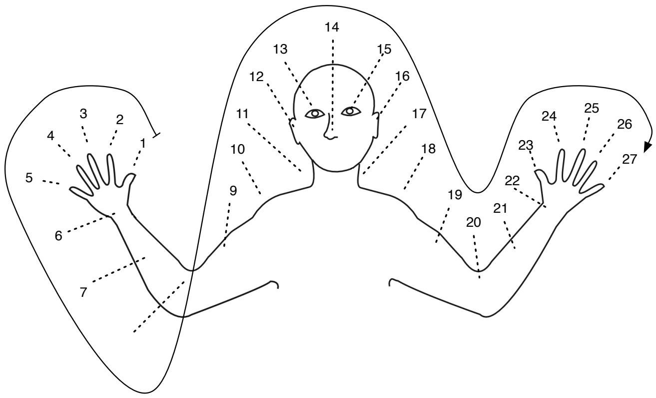
Some Notes on Research on Number Development in Cultural Practices

A third strand of research does not enter in any direct way the chapters in this section, but from my perspective is important in interpreting studies on mediation and numerical understanding. The work examines the interplay between children’s participation in cultural (or collective) practices and the development of numerical understanding (cf. Saxe, 2012). It is in cultural practices that numerical problems emerge and are conceptualized, and mediational forms are acquired or invented. So, a focus on practices would seem an important part of the puzzle in making sense of the role of mediational processes in the development of numerical understandings.

Some cultural practices targeted by researchers include family interaction related to the play of number games with toddlers (Saxe, Guberman, & Gearhart, 1987), children running errands to purchase goods for families living in Brazilian shantytowns (Guberman, 1996), children purchasing goods after school at small markets/liquor stores in the United States (Taylor, 2009), and children selling trinkets to tourists in the streets of Oaxaca (Sitabkhan, 2012). In these practices, children acquire and invent mediational forms, and they construct numerical ideas in communication about number and in numerical problem solving.

To illustrate Vygotskian and Piagetian themes in research that address the issue of the situatedness of mediation in practices, I consider some of my own work conducted with the Oksapmin, a remote cultural group in Papua New Guinea (Saxe, 2012). The Oksapmin traditionally use a 27-body part counting system for number representation (see Figure 1). I was interested in both the functions of the number system in traditional practices, as well as shifts in its functions over historical time, with the introduction of a money-economy and Western-style schooling.

Figure 1. Oksapmin's 27 body part counting system



First, some context: In traditional life in Oksapmin communities, people did not use the system for the purpose of arithmetical computations; in fact, there was no evidence that people engaged in such computations at all. With the introduction of schooling, however, children were engaged with a Western-styled curriculum of which arithmetic was a part. My interest was how children might be representing and making sense of the problems, given that instruction was in English, a language that children were only just learning. I expected that children might be using the 27-body part counting system. Observation and interview studies revealed that children were creating new ways of using the body system to solve arithmetical problems as a function of participating in mathematics classrooms. The new mediational approaches were not taught in school – teachers were foreigners and didn't know the system. Rather, children invented the new forms of mediation. An example of a new mediation function for the body system is evident in a child's solution of 16-7. To solve the problem, the first enumerates body parts to ear (16) and then counts down from the forearm (7) through the thumb (1) as he counts down from the ear (16) to the shoulder (10), leaving biceps (9) as the remainder. A video of this procedure is available at the link: http://www.culturecognition.com/fourth-grader-solving-16-7-body-system.

In the Oksapmin children's solutions we can find themes reflected in both Vygotsky's treatment of mediation. From a Vygotskian perspective, one can note the use of the cultural form of the body system to conceptualize and solve numerical problems. At the same time, from a Piagetian perspective, one can note children's construction of new kinds of one-to-one correspondences between body parts as they produce novel solutions to arithmetic problems.

What is both important and intriguing about the practice perspective on mediation is that one should attend to the functions that cultural forms serve for individuals as they structure solutions to recurring problems in daily life. From this perspective, without considering such practices it is problematic to draw conclusions about why one group (whether age or cultural group) shows differential cognitive developments. In the chapters to follow, often sidestepped in analyses are the practices in which children use mediational forms to address emergent problems in everyday life or in school. From my perspective, such a focus would shed light on effects like those documented.

The Chapters

As I have argued, the chapters in this section of the volume capture and extend contemporary strands of research that have roots in Vygotsky's and Piaget's early writings. In particular, the chapters engage themes that have historical grounding in both Vygotsky's focus on mediation with its cultural roots, and Piaget's focus on cognitive structures manifest in children's developing numerical ideas.

In Okamoto's chapter, she presents a thoughtful review of research on cross-linguistic differences in children's conceptualization of numbers, building on the Miura study (she was a co-author on a number of these studies). She extends the discussion by asking whether the documented effects found in Miura's work may be general – affecting mathematical development across many mathematical ideas and domains – or whether the language mediation effects are specific to closely related ideas, limited for example to the multiunit structure of number systems and place value. She reviews some of the contemporary thinking about numerical estimation without counting, and considers these capacities as resources that all children bring to emergent problems of quantification. She ends with a reasoned position that the linguistic effects are specific to a particular array of mathematical ideas.

In Towse's chapter, he takes up two issues related to mediation and numerical understanding. The first is a critical examination of Miura and colleagues original findings related to differences in the way that languages encode number and their implications for children's developing conceptualizations of number. Towse points out that Miura admirably crafted her study as an experiment of nature: She noted that languages differ in their alignment between the structure of their number word registers and the structural features of the written system of Hindu-Arabic numerals. One of Miura's insights was to take advantage of this "natural experiment" by contrasting different linguistic groups on a clever task that captured students' multiunit interpretations of number words. But Towse also raises cautionary notes. He is concerned about attributing a causal link from language to children's conceptualizations of number. To support his argumentation, he describes some well-crafted experimental studies that probe the causal link, showing that with an experimental paradigm, English-only speaking children can be induced into performing like their peers in East Asia.

In a second contribution to issues of culture and number, Towse considers the ongoing strand of research on the "mental number line" and ways that children position numbers on the line or label indicated positions with numbers. Among the questions that Towse brings forward are whether and in what way children's representation of numerical distances between numbers can be described with logarithmic or linear functions. In particular, his concern is with how children of the same ages or numerical skill levels differ with respect to their number representations. Again, the designs are clever. Building upon cross-cultural research by Siegler and others, Towse both documents cross-cultural differences in the shift to a linear representation, as well as evidence that is inconsistent with the claim that the shift drives early arithmetical developments.

In the chapter by Nunez & Marghetis, the authors focus on mediational shifts in development but of a different sort than number word systems alone. They argue for a conceptualization similar to Vygotsky's two lines of development, one "natural" and the other "cultural." Among the processes subsumed under a natural or biological line, they include the apprehension of small numbers (through "subitizing") and varied kinds of numerical discriminations through perceptual mechanisms. But in their mediational focus they point to some fascinating possibilities previously explicated in Nunez's book with Lakoff (Lakoff & Nuñez, 2000). Their focus is on forms of mediation involved conceptual metaphor and fictive motion: two important cognitive mechanisms that , in their view, mediate children's construction of a numerical world. The chapter presents some compelling argumentation that extends Vygotsky's cultural line of development in some very interesting and thought provoking ways.

The chapters close with a contribution from Sturman. The chapter has a different focus from the others. The concern is with two well-known international assessments of mathematics achievement, TIMSS and PISA. Sturman presents a thoughtful review of interpretive issues in making sense of cross-national differences in mathematical performance. In doing so, Sturman examines both the cultural and educational factors that can affect mathematical performance, and the methodological and conceptual difficulties in assessing international differences in attainment and in interpreting the results of such assessments. Thus, the section concludes appropriately with an emphasis on the complexity of assessing national differences in mathematics, and the dangers of overly simplistic interpretations.

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