

Book Review: When Oksapmin number system met Western arithmetic - Answering the question of how cognitive and cultural change induce each other.

Saxe, G. B. (2012) *Cultural Development of Mathematical Ideas. Papua New Guinea studies* Cambridge, UK: Cambridge University Press. 393 pages. Hardback: ISBN: 9780521761666. Paperback: ISBN: 9781107685697 Online ISBN: 9781139045360 Book DOI: <http://dx.doi.org/10.1017/CBO9781139045360>. £64 DOI:10.1017/CBO9781139045360.%20£64 [UK]

Anna Sfard

Published online: 31 December 2014
© Springer Science+Business Media Dordrecht 2014

What is the relation between individual cognition and culture? This question has always been on Geoff Saxe's mind, serving as the compass for his lifelong professional travel. The quandary is not a small one, to be sure, and not a new one either. Over centuries, it was pursued by the best of philosophical and scientific minds. So why tackle the old quandary again? Simply because we have never been sufficiently satisfied with whatever answer popped up. In the late 1970s, the question of how individual thinking and societal cultural forms interact with one another seemed as fresh as ever and vexing enough to start Saxe, at that time a young psychologist from the City University of New York, on a trip to the far away land of Papua New Guinea. With time, this proved to be the beginning of his remarkable lifelong friendship with this land and its people, as well as with several of his own collaborators and travel companions¹.

Why Papua, of all places? Probably because “in the realm of culture, outsidership is a most powerful factor in understanding” (Bakhtin, 1986, p. 7), and the Melanesian island was the place where the young American psychologist could feel as far removed from anything familiar as one could imagine (of course, this latter fact, in itself, could serve as a sufficient incentive: Saxe's was to be the kind of adventure most young Americans could only dream about). The first encounter with inhabitants of the two valleys known as the land of Oksapmin

¹On his early trip, Saxe was accompanied, among others, by Maryl Gearhart, who later became his companion in life. His other research and travel companions, each of whom participated in one trip, were Tom Moylan and Virginia Guilford, at that time graduate students working on their dissertations in anthropology, Indigo Esmonde, Saxe's PhD student, and Josh Saxe, Geoff's son.

A. Sfard (✉)
Faculty of Education, The University of Haifa, Carmel Mountain, Haifa 31905, Israel
e-mail: sfard@netvision.net.il

reassured him that he was looking in the right place. In 1978, having just completed a series of studies on counting practices of young American children, Saxe became fascinated with the unusual body-part counting system of the Oksapmins, who were using not only their fingers but also wrists, necks, forearms, noses, and armpits the way Americans were using such words as *four*, *six*, or *twenty-seven*. Just like English number words, body parts and their names served them as intermediaries in comparisons of sets.² Sometime later, seeing the body-part counting practices changing before his eyes in response to an influx of westerners and their practices, Saxe realized that in spite of the specificity of this local phenomenon, he was facing the rare opportunity to engage with his favorite general quandary. After all, what was happening to the locally developed discourse of quantities when another, evidently more effective form of communicating came its way, was a classic case of cognition being shaped by culture—and of the culture being molded in return.

The first research expedition to Papua New Guinea was followed by the second just 2 years later, in 1980. It took another 20 years before Saxe, by now professor at the University of California in Berkeley, traveled to his special research site for the third time. On this 2001 visit, he was armed with two decades' worth of addendum of experience, knowledge, and ideas. Over the years, his thinking had not just expanded—it changed. Indeed, it had been subject to sociocultural shaping no less than that of his informants. The new conceptualizations brought about new insights. Saxe's 2012 award-winning book³, *Cultural Development of Mathematical Ideas*, which invites the reader to relish all these riches, testifies to the value of focused, continuous, year-long intellectual effort. In our times of the constant feverish chase of novelty, where bandwagons seem the researchers' favorite means of transportation, this kind of project became a precious rarity. In this review, I hope to be able to convince the future reader of this unique book about the special power of lifelong intellectual fidelity to one inexhaustible source of wonderment.

1 Saxe's story

In launching his Melanesian project, Saxe has joined the long tradition of studies aiming at identifying cross-cultural similarities and culture-induced differences in how people act, think, and develop. This line of research, which emerged at the beginning of the last century and has been alive and well, albeit to varying degrees, ever since, is well known to contemporary education researchers thanks to the seminal work of the “second wave” cross-cultural researchers: the developmentalists Scribner and Cole (1981); (see also Cole, 1996), the anthropologist Jean Lave (1988), and several mathematics educators, with Teresina Nunes (Nunes, Schliemann, & Carraher, 1993), and Geoffrey Saxe himself (1988, 1991), among others. It was this strand of studies that, after embracing and then becoming increasingly critical of Piaget's theory, sought inspiration in the writings of Vygotsky and eventually made a decisive contribution to the sociocultural turnabout in developmental psychology and beyond. This is also where the claim about the inherently situated nature of learning has its roots. The intention of the “first wave” developmentalists who opted for Pacific or African research sites was to test Piagetian claims about cross-cultural developmental invariants. Quite soon,

² It is beyond the scope of this review to present the details of this ingenious method. However, thanks to Saxe's former work, the Oksapmin body-part counting is probably well known today to most mathematics educators. Those readers who would like to learn more about it even before reading this book are encouraged to watch the tale-telling clips at Saxe's personal website.

³ The book has received awards from the Cognitive Development Society, from the American Psychological Association, and from the Society for Psychological Anthropology (American Anthropological Association).

however, the encounter with the far away reality convinced them of the exaggerated, or perhaps even outright misleading, nature of Piaget's assertions. They decided that the then-dominant vision of learning was mistaken, and not only about the proportions of the natural and the cultural ingredients in the mix that constitutes us as humans but also about what it means to learn, and where this thing known as *knowledge* comes from. After some deliberations, many of the cross-culturalists concluded that the very notion of knowledge as a "thing" that can be "acquired" must be dropped. They opted, instead, for the idea of learning as one's changing participation in historically-culturally shaped activities.

Saxe's project began at the time when experimental cross-cultural psychology reached "high-water mark" and the "'cognitive revolution' swept the discipline" (Cole, 1996, p. 99). In that period, there was no doubt any longer that culture, far from being a mere provider of finishing touches, is a major player in the developmental game. No longer in doubt about its central role, the researchers were left with the question of how genetics and culture interact in constituting us as human beings. Within the wide array of studies that competed in the diversity of their settings, Saxe's project stands out as one of the rare cases of a longitudinal investigation that documented the historical change in the form of a motion picture rather than in just one or two snapshots. In the search for an answer to the question of nature versus culture, Saxe has been focusing on topics that belong to the traditional repertoire of cross-cultural studies. While asking, along with his colleagues, about the nature of numerical activities in everyday life, he was particularly interested in the transformations of these activities that happen under the influence of cultural, social, and economic changes on the one hand and in response to schooling on the other. His own and other cognitive psychologists' interest in numerical thinking can be traced back to the work of Piaget and his followers. Early Piagetians opted for the study of numerical thinking because this form of thinking was considered as culture independent and, as such, as a perfect context within which to observe "universal" developmental scenarios in action. At the time Saxe was launching his project, the belief in universality was gone, but the interest in numerical thinking as one that can be found in any culture persisted. The "schooling" strand was stimulated by the work of Vygotsky and especially by his famous study with Luria in Uzbekistan (Kozulin, 1990).

His being a follower of this long tradition does not prevent Saxe from making highly innovative contributions. Some of his most original insights result from his sustained attention to the *reciprocity* of cognitive and cultural change. Unlike the majority of cross-cultural researchers, Saxe views the culture-cognition relation as reflexive and supplements the story of changes that flow from culture to cognition with that of transformations that go in the opposite direction. Considering the two types of alterations as co-constitutive, he believes that they can only be investigated in tandem, as aspects of a single process. It is the nature and mechanism of these mutually induced transformations that are the heart of the matter when Saxe poses three interrelated queries: While counting practices are developing, what is it that changes? How does the change happen? Why does it happen?

The question of how to go about the task of looking for answers, which requires much creative thinking in any kind of study, becomes an almost insurmountable challenge when it comes to cross-cultural research. Here, the investigator is faced with at least two difficulties. The first of them, known as the fly-on-the-wall dilemma, arises mainly in those studies that aspire to the title of naturalistic. The observer's dilemma is how to become truly "unobtrusive"—how to watch people going about their daily affairs the way they always do, without influencing them in any manner. The other challenge, the dilemma of "outsider," is a part and parcel of interviewing and of analyzing the transcripts. While trying to hold a conversation across possibly incommensurable discourses, the researcher faces the paradoxical requirement:

To understand her interlocutor, she has to make every effort to start thinking in the strange ways that constitute the very puzzle she is trying to crack. To attain this goal, she must, first of all, become an outsider to her own culture—she has to free herself from the grip of her habitus, thereby opening herself to that of “the other.” Saxe’s recognition of the inherently insoluble nature of these dilemmas, far from being a hurdle, serves him well as an incentive for incessant quest after further methodological refinements. The results of his struggles are remarkable on two accounts. First, he seems to have come a much longer way in overcoming the obstacles than did most of the cross-cultural researchers; second, as successful as he managed to be, he has never rested on his laurels. In spite of the persisting lack of satisfaction—or perhaps just because of it!—the methods he worked out for himself over the years deserve the attention of any person interested in studying human cognition at large and numerical thinking in particular.

Complaints about the outsider’s dilemma, sometimes quite bitter, have accompanied cross-cultural studies ever since the emergence of the sociocultural line of study. As recalled by Sylvia Scribner, whose inquiries involved Vai people in Liberia: “we were hampered by conditions of work in unfamiliar culture. I came home, convinced I needed to be a native to undertake a research program” (Scribner, 1997, p. 358). It is in the recognition of this inherent difficulty that she eventually opted for studying cross-situational cognitive transformations within her own culture. It is also for this reason that she issued the call “to place the study of naturally occurring activities at the center of cognitive inquiry” (p. 355). Saxe, who ignored Scribner’s former advice, readily embraced the latter. Much of his 1980 and 2001 Papuan data was collected by observing Oksapmin people engaged in everyday tasks, mainly those involving money transactions. It is remarkable that, over the years, even this seemingly straightforward method underwent a considerable evolution, with the nature of the transformation reflected in the change from “observation” to “interview.” This alteration resulted from Saxe’s constant effort to reconcile two contradictory requirements: that of refraining from any intervention so as to let the local business go on as usual and that of controlling the informants’ activities in one way or another in order to ensure the emergence of the desired type of data. His eventual preference for interviews does not mean that he stopped being suspicious of this technique: “From the outset, I was concerned about threats to validity using interview method,” he says. This concern persisted in spite of his “efforts to mitigate validity threats” (p. 122). Once skeptical, he remained skeptical till the very end of his project: “Whether the interviews captured the key elements” of the investigated phenomena “merit(s) attention” (p. 122).

When it comes to *analyzing* the data, the dilemma of the outsider returns with vengeance. Faced with their notes and videos, how can analysts shield themselves against interpretations that come disguised as truths communicated by the world itself? How can they act against the “self-evident” that blinds them to the very possibility of different interpretations and may ultimately block their sense-making efforts altogether, leaving them more confused than they had been before having their data? In short, how can one bootstrap oneself out of his or her own understandings? The first rule, it seems, is to avoid immediate, on-the-spot interpretations as much as possible. Here, slowness and hesitance are virtues. You have to distance yourself from what is being said and done to be able to see the words and the deeds, as opposed to seeing just your own interpretations thereof. To make all this possible, you need to work with records of events—video clips and transcripts—that allow you to revisit all that has happened as many times as you wish. If there is one thing in Saxe’s admirable book I wish to complain about, it is the scarcity of the *verbatim* records of things said by either the researchers or the participants of their studies. Reported speech is preferred in this project over the direct. This kind of data leaves me with some important questions unanswered. For instance, when Saxe

says that the interviewer “asked the child whether the two men counted to the same number of sweet potatoes or whether they counted to a different number of sweet potatoes” (pp. 247–248), I wonder whether the term “the same” has actually been used. And when the author says that “a child claimed that they counted to different numbers” (p. 248), I want to know in what terms the child has expressed this claim. As I have learned from studies carried out in other countries, my own among them, small children never actually use the words *amount* or *number*, and they have particular difficulty with interpreting the expression *the same* in the context of numbers (Sfard & Lavie, 2005). Moreover, in spite of their ability to use number words such as *three* or *seventeen* in counting and comparison tasks, children treat these words as mere labels rather than as names of self-sustained mathematical objects. Whether the young learners do or do not “objectify” numbers impacts the way they interpret the interviewer’s questions. Saxe seems to be aware of all this when he says that “(t)hese problems (and the interview practice itself) may have been so foreign that what was being asked of participants was unclear to them” (p. 115). And yet, the absence of verbatim transcripts of the interactions makes it impossible to reconstruct interviewees’ attempts at interpreting the tasks and precludes the possibility of accounting for the “strange” things they might have said. All this leaves me with the wish to email the author of this great book with the request: Would you, please, go to Papua New Guinea once more and bring the kind of data that would make us, your readers, able to participate in the fiesta of sense-making?

Having said all this, I hurry to add that the rather thick ethnographical descriptions that one does find all through this book, accompanied by ample quantitative data, constitutes a solid ground for tackling Saxe’s *what*, *how*, and *why* questions. Telling the answers offered by the author himself would be a spoiler. I will thus restrict myself here to a highlight or two, hoping to be able to convey the general spirit of Saxe’s insights.

The question of what it is that changes in microgenetic (moment-to-moment or day-to-day, individual), ontogenetic (across one’s life span), and sociogenetic (in culture, on a historical timescale, collective) transformations is conceptual in nature. It must be answered in such a way as to make it possible for us to account for all the phenomena that attract attention. Unlike behaviorists and cognitivists, for whom the answers would be different depending on whether one speaks about micro- or ontogenesis, on the one hand, or sociogenesis, on the other, Saxe satisfies himself with a single entity that, for him, constitutes the object of all these changes: “cultural *forms* of representations and the *functions* they are used to serve in collective practices” (p.295; emphasis added). In the case of body-part counting system, these are arms, necks, and noses that correspond to the term *form*, whereas counting and calculating routines are what Saxe has in mind when speaking about *function*.

In response to the question of *how*, Saxe speaks about the cyclic process of shifts in form that lead to changes in function which, in turn, cause a new transformation in form, and so on. Interestingly, it is this kind of gradual change that resulted from the encounter between the Oksapmin counting system and the imported methods of Western arithmetic. Rather than supersede the traditional, body-based routines of comparing discrete sets and continuous quantities altogether, the formal, “globalized” numerical discourse and the related monetary practices led to a cascade of modifications in the use of body parts and in the vocabulary of quantities. In the dance of form and function documented by Saxe, the original patterns of acting evolved quickly toward the transplanted patterns, passing through a series of short-lived transitional forms and functions.

All this does not yet provide us with an answer to the *why* question—it does not say what it is that triggers the changes in the form-function relation in the first place. Saxe rejects the folk wisdom according to which necessity is the mother of invention. Instead, he ascribes central importance to local, mostly accidental modifications whose survival and dispersion, whether

synchronic (in space) or diachronic (along history), depend on regulative mechanisms that “constrain and enable the emergence, reproduction, and alteration of form-function relations” (p. 308). And he summarizes: “Representational forms and functions are reproduced and altered as the unwitting consequences of people working to accomplish local goals in communicative and problem-solving activity” (p. 194). Whether the modified tools and uses die quickly or survive somewhat longer hinges mainly on the question of how helpful they turn out to be in collective everyday practices.

2 Beyond Saxe’s story

Saxe’s Papuan expeditions were much more than ethnographic adventures, and the story of body parts turned numbers is much more than a curiosity. *Cultural Development of Mathematical Ideas* makes all this abundantly clear. The main contribution of the Oksapmin study, as I see it, is in the opportunity it creates for tackling the question of how it happens that individual “cognitive leaps” propagate in space and time, amounting to cultural changes, visible on the societal-historical scale. This question is of paramount importance, because the capacity for transforming our “forms of life” from one generation to another may well be this one feature that captures the essence of humanness and makes it possible to distinguish between people and other species in an unequivocal manner. Indeed, this capacity for societal-historical learning—for modifying typical recurrent ways of acting while making them more and more complex—may well be what defines us as human beings (Sfard, 2015). The Oksapmin study provides a direct glimpse into the mechanisms of historical transformations and offers more than a hint about how innovations spread and aggregate.

Saxe had the good fortune to see the substantial historical change happening before his eyes, within the span of his professional life. This is unusual. Most such processes last for centuries, if not millennia. The rapidity of the transformation observed in the Papuan study is easy to explain, though. The process of change in Oksapmin numerical discourse was mainly exogenous: Rather than being initiated entirely “from inside,” as a result of the Oksapmin people’s own initiative, this process was catalyzed, to large extent, by a transplant from another culture. The presence of the model to follow saved the considerable time needed for the primary invention to happen and hastened change. This research has also demonstrated that practical activities play a central role in the growth of discourse in general and of numerical discourse in particular. As shown in Saxe’s detailed analyses of the process of change, the attempt to graft Western arithmetic onto Oksapmin culture might not have succeeded as quickly and painlessly as it has, if not for the immediate relevance of this form of communication to the Oksapmin practices of economical exchange.

This conclusion has been recently reinforced by the findings of yet another, more recent cross-cultural research, in which I was involved. The study, conducted by Noah Morris in the Polynesian country called Tonga (Morris, 2014), has shown that, in the absence of practical activities likely to benefit from an imported discourse, this foreign way of communicating may not take root easily, if at all. In Tonga, this was the case with discourses on fractions and on probability, which are taught in Tongan schools and universities, but yield forms of acting quite different from those usually seen in English-speaking countries and lead to achievements much more disappointing than those attained by Tongan students in algebra, geometry, or analysis. Morris explains these findings by the fact that the traditional Tongan ways of life do not involve any activity requiring the use of parts of objects (Tonga people live in the Platonic world of whole objects only, preferably used in pairs) or tasks that invite predicting future events and assessing how certain one is of such prediction. By showing the implication of these absences for the development of discourse, Morris’s findings have demonstrated that the

existence of practical activities in which a transplanted discourse may be applied is more than just a favorable circumstance. Indeed, it constitutes a *necessary* condition for the successful acceptance of this discourse.

I would not end this review without a remark about the relevance of Saxe's study to the audience of this journal. I wish to claim that his Oksapmin research, although concerned mainly with issues of sociogenesis, has important implications for the learning and teaching of mathematics. As made clear in Saxe's analyses, and corroborated in Morris's study, new forms of communication do not catch on just because of being available. Two are needed for the tango of cultural change, and in the present case, practical activities are the necessary partners of discourses. This is true for individual learning no less than for historical development. In school, therefore, new discourses should be introduced in the context of familiar practical activities *that they are going to refine and, eventually, transform*. In the case of numerical discourse, this would mean, for instance, beginning in the first grade with asking "Which bag of candies do you choose?" rather than with the traditional, but in the eyes of the child, not very meaningful query "How many candies are there in the bag?" Another strand of implications relevant to teaching and learning comes from Saxe's insights on the way discourses develop. This theme, however, must be left for another paper.

3 In conclusion

I can see at least three reasons why this book is a must read for every mathematics education researcher. First, as an inexhaustible source of insights about numerical thinking and its development and also about human thinking in general, it can be trusted to provide the interested reader with a hearty serving of food for thought. Second, it is not every day that one sees a longitudinal study capturing a historical change in the making. This turns Saxe's story into something truly unique and precious. Finally, these 330 pages of the well-written text are too enjoyable a read to be missed.

Earlier in this review, fascinated with Saxe's narrative, I expressed my longing for yet another episode in the Oksapmin saga. It is time to admit, however, that even if Saxe does decide to undertake a new Papuan trip, one must be prepared for disappointment. Thirteen years after his last visit, he may have considerable difficulty trying to find Oksapmin people who are still counting on their bodies. But even if this prediction is true⁴, their ingenious, centuries old invention is not going to disappear altogether. Thanks to Saxe's work, the unique story of arms, necks, elbows, and noses turned numbers will stay with us for as long as we wish, to be retold and learned from over and over again. This story is rich enough to merit many happy returns.

References

- Bakhtin, M. (1986). *Speech genres and other late essays* (V. W. McGee, Trans.). Austin, TX: University of Texas Press.
- Cole, M. (1996). *Cultural psychology: A once and future discipline*. Cambridge, MA: The Belknap Press of Harvard University Press.

⁴ It is only after completing this review that I learned about Saxe's fourth visit to Oksapmin, which Saxe undertook earlier this year without waiting for my encouragement. In the light of this, he would now be able to either confirm or refute my prediction. To know which is true, let us wait for a sequel to his present book.

- Kozulin, A. (1990). *Vygotsky's psychology: A biography of ideas*. New York: Harvester Wheatsheaf.
- Lave, J. (1988). *Cognition in practice*. New York: Cambridge University Press.
- Morris, N. (2014). Probability, uncertainty and the Tongan Way. In P. Liljedahl, S. Oesterle, C. Nicol, & D. Allan (Eds.), *Proceedings of PME 38 / PME –NA 36* (Vol. 4, pp. 241–248). Vancouver, Canada: PME.
- Nunes, T., Schliemann, A., & Carraher, D. (1993). *Street mathematics and school mathematics*. New York: Cambridge University Press.
- Saxe, G. B. (1988). Candy selling and math learning. *Educational Researcher*, 17(6), 14–21.
- Saxe, G. B. (1991). *Culture and cognitive development: Studies in mathematical understanding*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Scribner, S. (1997). In S. Tobach (Ed.), *Mind and social practice: Selected writing of Sylvia Scribner*. New York: Cambridge University Press.
- Scribner, S., & Cole, M. (1981). *The psychology of literacy*. Cambridge, MA: Harvard University Press.
- Sfard, A. (2015). Learning, commognition and mathematics. In D. Scott & E. Hargreaves (Eds.), *The Sage handbook of learning*. London: Sage. (in press)
- Sfard, A., & Lavie, I. (2005). Why cannot children see as the same what grown-ups cannot see as different?—Early numerical thinking revisited. *Cognition and Instruction*, 23(2), 237–309.