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CHAPTER

9

Emergent Arithmetical Environments in the Context of Distributed Problem Solving: Analyses of Children Playing an Educational Game

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The view that cognition is inherently situated—linked to the cultural practices in which individuals function—is common to recent sociocultural treatments of cognition (e.g., Cole, 1990; Greeno, 1991; Hutchins, 1991; Lave, 1991; Wertsch, 1991). From the sociocultural perspective, cognition—whether the mathematics of candy sellers in urban Brazil (Saxe, 1988, 1991), the blueprint/scale knowledge of construction foremen (Carraher, Schliemann, & Carraher, 1988), or the engineering knowledge of electrical technicians (Janvier, Baril, & Mary, 1993)—takes form in relation to a range of social and cultural processes, including the particular artifacts and tools that are valued in practices, power and role relations that emerge and become institutionalized in practices, and social interactions with others. From the sociocultural perspective, knowledge as manifested in practices is culturally mediated and socially shared (Cole, 1991).

Sociocultural views of cognitive functioning pose interesting conceptual and methodological challenges for the study and analysis of children's learning and cognitive development. We need conceptual models of cognitive development that can be used to organize analyses of cognition in situ—models that reflect intrinsic relations between the constructive, sense-making activities of the individual and sociocultural life. We also need methods for observation and analysis that extend such conceptual models into the field—methods that reveal the culturally textured character of cognition as it emerges in people's daily practices.

This chapter sketches a general framework for the study of children's learning environments informed by sociocultural perspectives of cognitive development. Central to the framework is the construct of emergent goals (Saxe, 1991). Emergent goals serve as a basis for both the analysis of children's construction of cognitive environments in practices and a conceptualization of children's learning. Guided by the framework, we explore questions about children's dyadic play of an educational game in their classrooms: Under conditions of dyadic activity, how can we understand the learning environments that are taking form for individuals?

Our efforts to extend the *emergent goals* framework to methods for the analysis of children's dyadic play have led to intriguing problems in coordinating analyses of what tasks the dyad is solving and the emergent goals that individuals are constructing and accomplishing in their joint activity. We describe two types of data, each of which addresses these issues. One type involves a case-by-case analysis of videotaped excerpts of joint play. The other involves the aggregation of case-by-case analyses through a framework-based coding scheme. In concert, these techniques provide a means of revealing general characteristics of the relations between the cognitive work that dyads accomplish as a unit and the cognitive environments that emerge for individuals.

THE GAME

ED - ? game of Treasure Hunt is de-

The educational game, Treasure Hunt, is depicted in Fig. 9.1. The game was developed by the UCLA Peer Interaction Group[1] to serve two functions. First, it was designed as an enrichment activity: It was an effort to bring insights from field studies of mathematics learning in daily practices into the elementary school classroom (see Saxe, 1995, for a discussion). Second, the game was also designed to support our analysis of the mathematical environments children construct in play. In developing the game, we made sure that the principal artifacts that children manipulated for number representation were easily identifiable by an observer and recordable on a video camera and that the game supported children's verbal interactions about their math-linked activities.

[1]The UCLA Peer Interaction Group has included Joseph Becker, Teresita Bermudez, Kristin Droege, Tine Falk, Steven Guberman, Marta Laupa, Scott Lewis, Anne McDonald, David Niemi, Mary Note, Pamela Paduano, Laura Romo, Geoffrey Saxe, Rachelle Seelinger, and Christine Starczak.

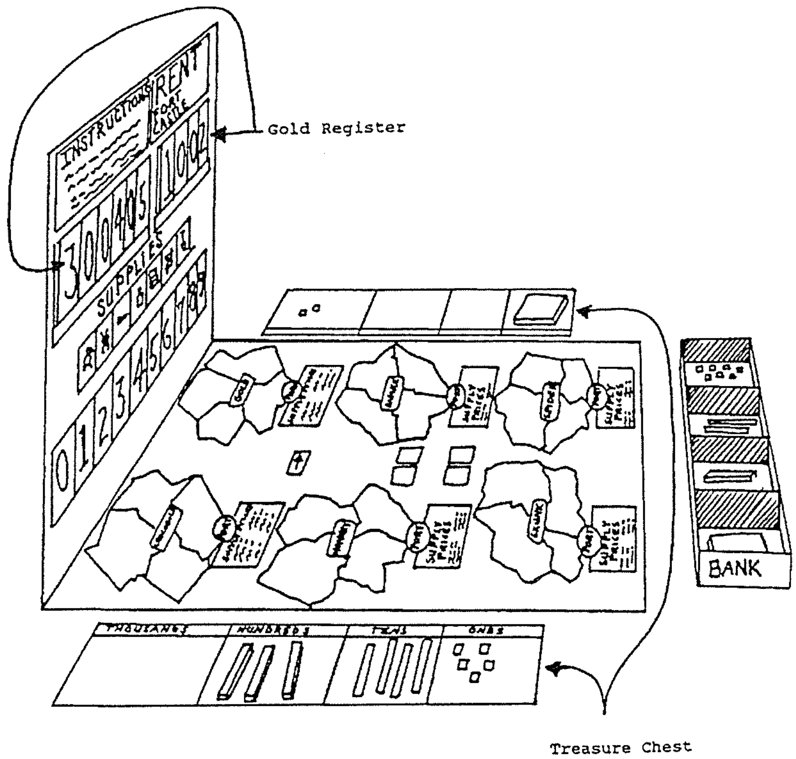

FIG. 9.1. The Treasure Hunt game.

Children play Treasure Hunt in pairs; the prescribed objective is for each child to acquire more gold than the other. To this end, children assume the roles of treasure hunters in search of gold doubloons—gold painted base-10 blocks in denominations of 1, 10, 100, and 1,000 units. Children store their gold in treasure chests that consist of long rectangular cards organized into thousands, hundreds, tens, and ones columns, and children report the quantity of their gold on a gold register with printed numerals. The child who acquires the most gold wins the game.

Children alternate rolling a die on a large rectangular playing board that consists of six islands. As a function of the roll of the die, players move to new islands, purchase supplies at the island ports, and then move, with the draw of a colored card, to one of four geographical regions on the island. At the region, players receive messages that offer opportunities to use their supplies either to gain additional gold or to protect their existing gold. (An enlargement of Snake Island—its ports and geographical regions—is contained in Fig. 9.2.) At the end of each turn, players must report the quantity of gold in their treasure chests by placing numerals on their gold register.

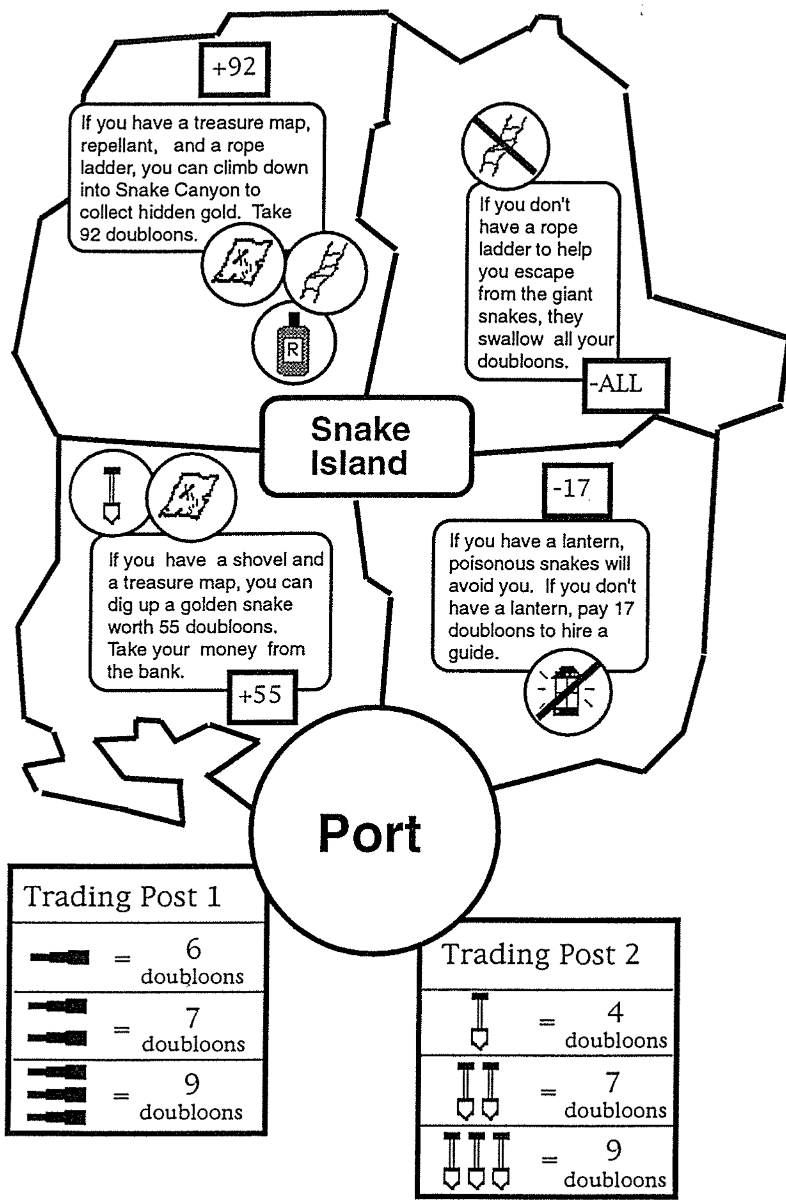

FIG. 9.2. An enlargement of Snake Island.

Players are subject to challenges from their partners for inaccurate gold register reports (e.g., reporting 9 hundreds, 8 tens, and 15 ones [9(100) 8(10) 15(1)] as 9,815). In our study of children's play, we observed and videotaped 32 dyads, including 16 third- and fourth-grade pairs, 8 third-grade pairs, and 8 fourth-grade pairs. Participants were children attending an inner city school in Los Angeles.

THE EMERGENT GOALS FRAMEWORK

The emergent goals framework was developed in prior work on children's learning in cultural practices (Saxe, 1991). Central to the framework is the view that children create learning environments through their construction of goals. Goals are not static forms that exist ready made in the minds of subjects. Rather, goals emerge as children bring to bear their own understanding to organizing and accomplishing problems that emerge during their participation in cultural practices. We designed Treasure Hunt to simulate a cultural practice—that is, we provided norms of interaction (e.g., turn-taking rules) and artifacts (e.g., gold blocks). As we discuss later, children's goals are influenced not only by the structure of the game but by their own prior knowledge, thematic role play, and sense-making efforts.

The emergent goals framework consists of three components, each of which takes as its starting point the centrality of goals in cognitive development. The first component is concerned with the analysis of how goals emerge in practices: the way children's active sense-making efforts become interwoven with sociocultural processes in their construction of goals. The second component is concerned with cognitive development—the dynamic interplay between cognitive forms and functions as children construct ways of accomplishing emergent goals (Saxe, 1991, 1992; Saxe, Guberman, & Gearhart, 1987). The third component is concerned with the interplay between cognitive achievements across practices—children's efforts to appropriate understandings from one practice to accomplish goals in another practice. In this chapter, we limit the analysis to the first component: how goals emerge during children's joint play of Treasure Hunt.

The Analysis of Children's Emergent Arithmetical Goals in the Play of Treasure Hunt

In their play of Treasure Hunt, children form a wide range of goals. Some involve the construction and implementation of strategies to win. Others involve making a friend of their partner. Still others involve efforts to help their partners understand how to accomplish a computation. For the pur-

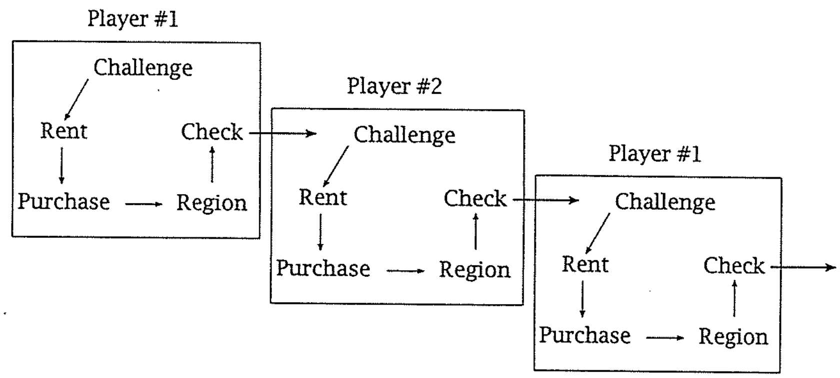

FIG. 9.3. The five-phase turn-taking structure of play. Each square (consisting of challenge, rent, purchase, and check phases) represents a single player's turn.

poses of this chapter, we focus strictly on the arithmetical goals that emerge in the web of children's concerns and motives in play.[2]

To analyze the emergent goals in play of Treasure Hunt, we focus on four parameters, each of which is implicated in the process of goal formation. The parameters include activity structures, social interactions, artifacts and conventions, and prior understandings. Below we extend the four-parameter model developed in prior work (Saxe, 1991) to the analysis of Treasure Hunt, pointing to how the model frames our analysis of children's emergent goals in the context of distributed problem solving.

Parameter 1: Activity Structures. The structure of Treasure Hunt consists of five phases in a turn-taking organization—an organization that supports the emergence of a wide range of problems. The structure is depicted in Fig. 9.3 and described in detail elsewhere (Saxe, 1992). Within this general structure, children's purchase of supplies is a principal phase of the game that could emerge in every turn; it is this phase during players' turns that provides the context for our later analyses. During the purchase phase, players consult the supply menus posted at the islands' trading posts (see Fig. 9.2) and select supplies to buy.[3] The purchase phase substructure sets

[2] In this chapter, we use the construct of arithmetical goals (and subgoals) to include both the conscious objectives that individuals create as they are accomplishing emergent problems as well as the less than conscious arithmetical constraints that individuals satisfy in the course of their activities to accomplish arithmetical problems. In forthcoming work, this distinction is elaborated in a discussion of the emergent goals framework.

[3] The supplies take on significance during the next phase of the player's turn when the player draws a color-coded card that directs him or her to one of the four geographical regions. At the region, the player receives a message indicating whether he or she may trade some of the specified supplies to either gain gold or avoid losing gold.

the stage for the emergence of various kinds of arithmetical goals. For instance, players often choose to buy a number of supplies—choices that lead them to add or multiply supply values and then subtract the sum from their gold.

Parameter 2: Artifacts and Conventions. During a purchase, children's mathematical goals are interwoven with the principal artifacts of the game. These include the price-ratio menus used at island ports, base-10 blocks (see Fig. 9.4), and the numerals for representing gold on the gold register. These artifacts constrain and enable the emergence of particular arithmetical goals. For instance, in the purchase of supplies, players need to accomplish subtraction problems in the medium of base-10 blocks. Due to their physical characteristics and conventional significance, the base-10 blocks afford particular kinds of solution approaches. For instance, some emergent goals linked to paying for supplies require children to construct equivalence trades to achieve an adequate solution, as when children need to pay for supplies but do not have exact change. In such cases, players need to construct goals to produce one or more equivalence trades of a larger denominational doubloon [e.g., 1(100)] for smaller denominational doubloons [e.g., 10(10)].

Parameter 3: Prior Understandings. The prior understandings that children bring to Treasure Hunt have implications for the arithmetical goals that emerge during play. For instance, some children have difficulty understanding the denominational structure of the blocks. They may treat all blocks with a value of unity, not conceptualizing blocks of different size with reference to their many-to-one equivalence relations [e.g., 10(1) is equivalent to 1(10)]. As a result, when faced with a problem that requires payments when no exact change is available [e.g., paying 14 when one has only 9(100) 7(10)], children may structure subgoals in which the denominations of the

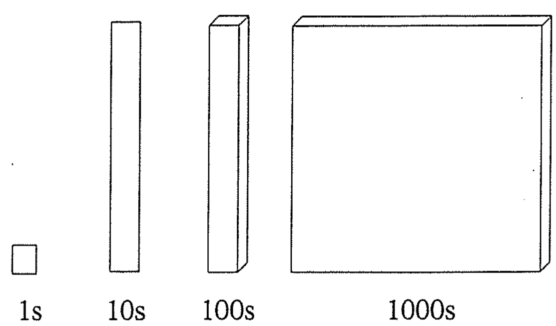

FIG. 9.4. Gold doubloons (gold-painted base-10 blocks) are one of the principal artifacts of play. Doubloon units include 1, 10, 100, and 1,000 pieces.

blocks are not respected in the formation and accomplishment of the arithmetical problems. For instance, a child who owes 14 doubloons may treat all blocks as unity and pay with 14 blocks of varied denominations [e.g., 7(100), 7(10)]. Thus, goals are rooted in children's conceptual constructions, and analyses of children's arithmetical environments in the context of distributed problem solving must be grounded in a treatment of children's prior understandings.

Parameter 4: Social Interactions. Children's goals often shift and take form as they participate in practice-linked social interactions. As a function of these interactions, children's goals may become reduced or elaborated in complexity. For instance, in children's play of Treasure Hunt, we noted two patterns of social interaction during the purchase of supplies, both of which served to sustain play and led to alterations in children's arithmetical goals and subgoals. In one, a player received direct assistance from another (often when the player encountered difficulty in accomplishing a problem). In the other, thematic roles emerged during the purchase of supplies that supported the less knowledgeable partner. In such interactions, one child typically played customer and the other storekeeper—a distribution of labor that often supported the solution of arithmetical problems and ensured continuity of play.

EMERGENT GOALS IN DISTRIBUTED PROBLEM SOLVING: METHODS OF ANALYSIS

We now turn to techniques to study children's play of Treasure Hunt. We point to two approaches derived from the four-parameter model. One technique is qualitative, focusing on individual cases. The other is quantitative, involving the development and application of a coding scheme that has permitted us to aggregate observations over turns, individuals, and dyads; measures were then constructed from codes and analyzed using inferential statistical techniques.

For illustrative purposes, we frame the discussion of qualitative and quantitative techniques with regard to two questions:

1. *What is the complexity of the arithmetical problem that the children accomplish as a dyad?* In our case study analyses, we specify problem complexity by detailing the denominations of doubloons that players have in their treasure chests and the quantity of doubloons the player must pay. In the aggregated analyses, we dichotomize problem complexity into two types: (a) problems that do not require denominational trades for an adequate

solution, and (b) problems that require one or more trades to produce an adequate solution.

2. *What numerical goals do individuals structure and accomplish in the context of the dyadic activity?* In both the case study and aggregated analyses, we make inferences about the goals and subgoals children structure–goals that emerge as the dyads accomplish emergent problems. Such inferences are generally based on an analysis of children's activity–for instance, when a child gives his or her partner one 100 block and asks for ten 10 blocks, we infer that the child generated a goal (or subgoal) to accomplish an equivalence trade of one 100 block for ten 10 blocks.

The two methods–the qualitative and quantitative–complement one another in the analysis of children's emergent environments in play. The analysis of individual cases is a means of gaining insight into the dynamics of distributed problem solving: How are the framing and accomplishment of problems *stretched over* (to use Lave's, 1991, term) individuals and artifacts in joint activity? The quantitative analysis of behaviors across cases is further removed from direct observation; however, it allows us to test general claims about processes of goal formation. For instance, we ask: How does variation in players' arithmetical understanding (as indexed by their grade levels) affect their formation and accomplishment of arithmetical goals in play? How are the formation and accomplishment of goals affected by the grade level of one's partner? With what regularity do particular types of interactions occur in dyads?

Analyses by Cases

We began our analysis of cases by observing instances of children's joint problem solving during the purchase phase of play. Through an inspection of video tapes, we found two prototypical forms of distributed problem solving. Within each form, we found variation in both the arithmetical environments children were structuring and the opportunities to construct more complex goal structures that the more competent players provided for their less competent peers.

In the case of direct assistance, the distribution of problem solving emerged through one player's initial decision to purchase a particular set of supplies. Subsequently, at some point in the solution process, the player had difficulty structuring the arithmetical subgoals–such as determining the total cost of to-be-purchased items, determining how to pay for the purchase–that would lead to an adequate solution. The process of goal construction and accomplishment then became interwoven with assistance from the partner, often under the press of the partner's concern to keep the game moving.

In the case of thematically organized assistance—the second type of distributed problem solving—the solution arose out of the thematic roles children invented: Some children came to assume the roles of storekeeper and customer in the purchase phase—roles that, on occasion, became institutionalized over the course of play. Thus, a customer might select the supplies to purchase and, in turn, a store keeper might then sum the cost of the supplies and tell the customer the purchase price. The customer might then produce a payment and the storekeeper the change.

Sometimes thematic roles and direct assistance became blended with one another in interesting ways. For instance, when there was distribution due to thematic role divisions, sometimes the storekeeper required assistance from the customer in formulating or accomplishing the arithmetical goals and subgoals of his or her role.

A Case of Direct Assistance. ERI and KEV, as a dyad, successfully accomplished many problems that required denominational trades in play. However, when looking at ERI and KEV's individual participation, we observed differences in the arithmetical goals that they structured in play. Through his choice of supplies to purchase, KEV often generated complex arithmetical goals—goals that required two and three equivalence trades of doubloons to be solved successfully. It was ERI, however, the older and more competent of the two players, who made it possible for KEV to purchase his supplies by constructing and accomplishing the subgoals necessary for KEV's purchases. Consider one example of this activity.

> KEV said he wanted to purchase one treasure chest and two bottles of insect repellent, which cost six and seven doubloons, respectively. KEV had difficulty formulating the arithmetical goals to add the quantities (6 + 7), and ERI quickly took over, determining that it would cost KEV 13 doubloons, plus 5 more for an earlier debt. KEV only had 900 pieces [9(100)], so his payment goal would require two trades if he were to pay with exactly 18 doubloons: 1(100) block for 10(10) blocks, and 1(10) block for 10(1) blocks. KEV, apparently unaware of the value of the blocks, asked if he needed to pay with all of his nine (100) pieces. ERI realized that a more appropriate subgoal was to subtract 18 from one of KEV's 100 blocks. She then proceeded to take 1(100) from KEV's treasure chest, put it in the bank, and determine how much change he should get. She covertly (mentally) performed the subtraction and appropriately gave him 8(10) blocks and 2(1) blocks in change. KEV then counted his doubloons, denomination by denomination, and changed the numerals on his gold register appropriately.

In this example, KEV initiated the construction of the arithmetical problem of paying 18 doubloons through his intention to purchase supplies, but it was ERI who structured and accomplished the necessary subgoals for the subtraction. Although ERI's contribution enabled KEV to remain involved and continue participating in the game, the way in which she accomplished the subtraction subgoals did not afford KEV access to the processes used to accomplish the higher order goal of paying for the purchase. For example, she did not (a) verbalize what she was doing, (b) explain that the subtraction could be accomplished through trading, or (c) break down the process into trades that KEV could witness. She did not even explain the concept of block value and block equivalence when KEV was about to pay with his nine 100 blocks. Although as a dyadic unit the children structured and accomplished complex arithmetical problems, their individual construction of arithmetical goals and subgoals differ. ERI constructed, composed, and decomposed values in play. In contrast, KEV was not engaged with similar arithmetical constructions. Nor did ERI provide KEV with access to her construction of goals and subgoals—access that may have served as a model for KEV's subsequent activities.

Thematic Role Divisions Leading to Distributed Problem Solving. VER and TON, a similar dyad in composition to KEV and ERI, provide an interesting contrast. Like KEV, VER, the less competent of the dyad, also initiated purchases, whereon TON constructed and accomplished the necessary arithmetical subgoals. However, unlike ERI, TON constructed these goals in a manner that granted VER access to the processes involved in solving the problem by overtly labeling, verbalizing, explaining, and restating her activities. In the following interaction, this access took form in the context of a thematic role division.

> During a purchase phase, VER decided to buy one map and one parrot at a cost of six and five doubloons, respectively. TON, in her role as storekeeper, performed the addition step by step, saying, "the map is 6 and the parrot is 5" and counting on her fingers, "6, 7, 8, 9, 10, 11. Okay, 11 doubloons." VER had blocks only in denominations of (100)s and (10)s, and therefore could not pay with the exact amount of doubloons. Instead, VER handed TON 2(10) blocks and asked for change. TON handed her the change by counting on, "12, 13, 14, 15, 16, 17, 18, 19, 20," like a cashier.

When it was TON's turn, VER assumed the role of the storekeeper; she was then responsible for performing the subgoals necessary to accomplish the higher order goal set by TON. Because she was less competent in math, VER sometimes ran into difficulties trying to accomplish these subgoals. When this happened, TON intervened and assisted VER, guiding her through the necessary processes. Consider another example:

> TON decided to purchase one lantern, one ladder, and a castle (composed of three rooms) and asked VER how much it all amounted to. Lanterns and ladders cost three doubloons each and the castle rooms cost four doubloons

> each. VER, in her role as storekeeper, proceeded to add out loud, "ladders cost three, three plus three six, and three . . . nine." She stated the price of a fort room (three doubloons) instead of a castle room, and also added the cost of only one room instead of the three rooms necessary to build a whole castle. TON, although in the role of customer, intervened and counted on her fingers (with a "counting on" strategy; Fuson, 1988), concluding, "18, you buy three of these" (referring to the castle rooms) "so that's 18 altogether." They discussed it for a short time, with TON explaining why the purchase totaled 18 doubloons. After reaching an agreement, TON stated, "I'll give you $18." She then attempted to pay with exact change [1(10) and 8(1)], but did not have enough ones pieces. She then asked VER, "Could I have change? Here's $20." VER took the 2(10) TON handed her, and said, "$20," followed by a long pause. TON then again assisted VER by telling her "two of these" and pointing to the ones pieces in the bank. VER then handed TON two doubloons in change.

In this example, TON became the customer and VER the storekeeper. The roles supported VER's engagement in the construction of arithmetical goals. Were it not for these thematic roles, perhaps VER would not have become involved in trying to determine the sum or the subtraction, as was the case for KEV. Indeed, this thematic role division gave VER opportunities to participate, solve problems, and create more complex mathematical learning environments—opportunities that KEV never received. During play, VER was able both to observe TON's actions when TON was the storekeeper and assume the storekeeper's role although she needed assistance from the customer to solve the emergent problems. Therefore, she was able to have access to the processes involved in the accomplishment of the subgoals by observing someone else perform them overtly and by having opportunities to attempt them herself.

A Child's Effort to Provide Access to Subgoal Construction

WEN and ANG showed a particularly interesting display of a more competent player attempting to provide her partner access to the construction of arithmetical goals and subgoals over the course of play. WEN, the older and more capable player, functioned in the role of storekeeper (when it was ANG's turn), performing all the additions and subtractions (i.e., determining the total cost and providing change). However, when it was WEN's turn to purchase supplies, WEN made her construction of subgoals quite accessible for ANG by always paying with the exact change, first performing the trades required to pay with exact change if necessary. Consider the following situations that display these forms of interactions:

> WEN had to pay ANG 50 doubloons in rent (for landing on a region in which ANG had a fort), but only had 3(10) [in addition to her 100s blocks]. Rather

than opting to pay with 1(100) and involving ANG in a subtraction problem (100 minus 50), WEN first traded 1(100) for 10(10)s and then paid ANG the exact amount with 5(10). During the purchase phases, she engaged in the same behavior. For example, she once had to pay 29 doubloons, but only had 4(1) [in addition to her other denominations]. Rather than paying ANG with 3(10), which would have required ANG to perform the subtraction of 30 minus 29, WEN traded 1(10) for 10(1) and paid ANG with exact change.

WEN's actions during her own turns was another way of providing a player access to the processes involved in payment/subtraction, much like TON's cashier style. Although WEN's behavior did not require ANG to perform the necessary subgoals, ANG was able to observe the steps required to successfully achieve a solution both when it was ANG's own turn (during which WEN determined the cost of ANG's purchase and change) and when it was WEN's turn. WEN's style of interaction proved to be an effective learning situation for ANG. As the game proceeded, ANG appropriated WEN's trading behaviors during her own turns. WEN apparently engaged in this trading behavior during her own turns (and not during ANG's turns) because of her belief that ANG would have difficulty performing the operations required to give change. For instance, when ANG started to perform trades during her own turns, WEN stopped paying ANG with the exact amount and started requesting change.

Remarks on Case-Based Analyses

The previous examples present cases in which dyads were accomplishing arithmetical problems involving the summation of purchase prices and the payment of doubloons that required denominational trades. For all the children, the framing and accomplishment of the problems were distributed over the dyad and the materials with which they were engaged. However, the character of the distribution differed. The less capable children tended not to structure goals that required them to conceptualize doubloon values in terms of equivalence trades. Indeed, when such problems emerged, they were distributed over the dyads in ways so that the less competent player did not need to structure or accomplish the equivalence trade. In contrast, the more competent children were structuring relatively complex arithmetical goals in their solutions.

We also observed marked differences in access to the construction of more complex arithmetical goals that the more competent member of dyads provided the less competent players. Sometimes greater access involved restructuring a problem context so that the less competent player could first observe and then accomplish higher level problem solutions later. Other times greater access was provided by the more competent player's verbal explanations that supported understanding the purpose of higher level goals. For some, but not all, dyads and for some turns but not others, the social interactions provided contexts for the less competent players to structure higher level arithmetical goals.

In our analysis of individual dyads, we used the four-parameter model to frame the analysis of emergent goals, focusing on the arithmetical environments that emerged for individuals in the context of distributed problem-solving activities. Although these analyses provide a portrayal of emergent arithmetical goals in the context of children's dyadic interactions, to make claims about what larger populations of children do in Treasure Hunt, or the way in which children's performance varies by their and their partners' grade level, we turn to our aggregated quantitative analyses.

AGGREGATED ANALYSES BASED ON CODING SCHEMES

We developed coding schemes based on the emergent goals framework to extend insights gleaned from analyses of individual cases to an analysis of emergent environments across dyads. We aggregated codes assigned to individuals and dyads and then analyzed these aggregated codes as a function of the grade levels of the players and their partners. We sketch a small set of these schemes here. The examples illustrate some of the problems and merits of an aggregated approach to the analysis of emergent goals in play. (A forthcoming publication will contain a more complete presentation of schemes.)

An Introduction to the Coding Schemes

In the construction of a coding scheme intended for the construction of quantitative measures of learning in practices, one grapples with tensions between a conceptual treatment of the phenomena under study and two principal constraints: (a) the ability of multiple coders to replicate one another's application of the scheme to a corpus of observations (in our case, videotaped records of play), and (b) the distributional requirements imposed by the use of particular statistical techniques. The potential payoff with the scheme-based approach is that one has a base of evidence that can be used to support more general claims than case study evidence provides.

In the analysis of Treasure Hunt, the creation of schemes was guided by the four parameters of the emergent goals framework. The five-phase turn-taking activity structure of the game (Parameter 1) led us to code children's behavior by turn; within each turn, we partitioned behavior into problems that emerged in each of the five phases (challenge, rent, purchase, region, check). The artifacts of play (Parameter 2), like the base-10 blocks, constrained the problems that emerged during the phases. We coded two types of problems linked to this artifact: problems that required denominational trades to achieve exact payments and those that did not. Our analysis of the various forms of social interaction (Parameter 3) linked to the emergence and accomplishment of problems led us to code whether the organization of the interaction was based on thematic roles or whether the interaction was one of direct assistance. Further, we rated on a 5-point scale the extent to which the player (the child whose turn it was) accomplished the particular problem-linked goals and subgoals of the problem on his or her own or with assistance from the partner. Finally, we represented children's understandings (Parameter 4) in play through our selection of subjects, including third and fourth graders in our sample of players (who also were screened via a math achievement test): The fourth graders' arithmetical understandings were more sophisticated than were the third graders'.

In reviewing the coding schemes and the findings that they yield, we revisit issues addressed in the qualitative analyses of emergent arithmetical goals. Framed by an initial examination of the extent to which two principal problem types emerged and were successfully accomplished in play, the analyses focus on differences in the character of children's emergent goals in the context of distributed problem solving as a function of the players' grade and the grade of the players' partner.

Buying Supplies: Accuracy by Problem Type

We saw in the case studies that players frequently bought supplies. The emergent arithmetical problems that issued from purchases vary in complexity as a function of the denominational distribution of gold pieces that players have in their treasure chests and the cost of the to-be-purchased supplies: Sometimes players have the denominations to pay for their purchase with exact change and other times they do not. On occasions when they do not have exact change, we see that some players, like ANG, have difficulty accomplishing an adequate trade of greater for lesser doubloon pieces.

On the basis of the case study analyses described earlier, we expected that children's ability to adequately accomplish purchases would vary by the complexity of the emergent arithmetical payment problems. To accomplish the aggregated analyses, we coded the character of the gold problems with which children were engaged during their turns. Supply purchases in which a player was able to pay with exact change were coded as *No Trade problems*; purchases for which players would have to change one denomination for another to make an exact payment were coded as *Trade problems*. To determine whether children were accomplishing problem-linked goals, we also coded the accuracy of children's solutions. For each problem type

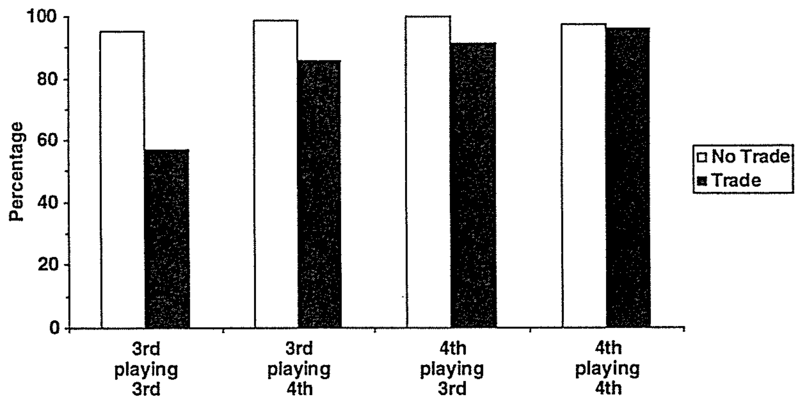

FIG. 9.5. Mean percentage of payment problems solved correctly.

we determined the percentage of accurate solutions for each child and then computed the mean of these individual percentages for each subgroup of children.

Figure 9.5 contains the results of the analysis—the mean percentage of payment problems that children solved correctly as a function of problem type, player's grade, and grade of the player's partner. The figure shows that when children were engaged with No Trade problems, they were usually accurate, regardless of their grade level or the grade level of their partners. Thus, in their play of the game involving payments, even the third graders playing third graders participated competently when No Trade problems arose, constructing goals of counting and composing single- and ten-unit doubloon pieces.[4] However, when children were engaged with Trade problems, group differences emerged. The third graders playing other third graders solved fewer of these problems correctly than did third graders playing fourth graders or fourth graders playing either third or fourth graders.[5] These findings suggest that, although such problems emerged for all children, the third graders playing third graders had difficulty structuring goals involving trades.

Thematic Division of Labor

Recall, as in the case of TON and VER presented earlier, that the social organization of making a payment could vary markedly especially in the context of payments involving trades. Sometimes roles emerged during

[4]The cost of most supply purchases ranged between 6 and 30 doubloons.

[5]Details on the statistical analysis, reliability of the coding schemes, and sampling issues are presented in a forthcoming monograph.

payments in which one child became the storekeeper and the other became the customer. When such thematic roles emerged, the players' work of constructing and accomplishing the payment problem became distributed over the dyad: The task of making an initial payment was given to the player (or the customer) and the task of making change (if any) became that of the partner (or the storekeeper).

We also noted that there may be an important function of this emergent social organization for mathematics learning. Under thematically organized divisions of labor, some third graders were able to participate in the construction and accomplishment of complex arithmetical problems, which they were unable to solve on their own. Indeed, in thematically organized divisions, third graders were constructing and accomplishing goals similar to those associated with their solutions to No Trade problems (making exact payments), although they were participating in the solution of the more complex higher order Trade problems (making equivalence trades).

To determine the extent to which thematic role divisions occurred and whether such divisions occurred more frequently for some groupings of dyads than others, we coded whether a thematic division of labor emerged for each purchase. We then determined the proportion of each player's Trade problems for which there was a thematically organized division of labor. Figure 9.6 contains the percentage of Trade problems for which children assumed thematic roles as a function of the player's grade and the grade of the player's partner. We included in the computation only those occasions that resulted in accurate solutions.

We see in Fig. 9.6 that when third graders played fourth as opposed to other third graders, more thematic role distributions emerged. This suggests that the greater success of the third graders paired with fourth graders as opposed to other third graders (as shown in Fig. 9.5) was due to the

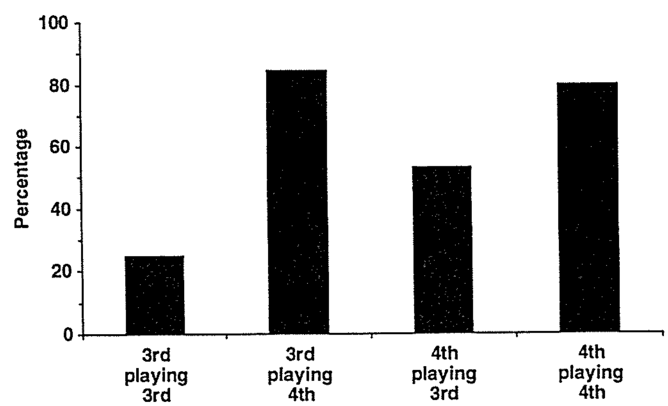

FIG. 9.6. Percentage of trade problems solved with thematic division of labor.

thematically based division of labor that emerged in these dyads. Also noteworthy is that, in mixed-grade dyads, the thematic role divisions occurred less frequently for fourth graders than it did for third graders. This drop in frequency suggests that division of labor was critical in third but not fourth graders' adequate performance on Trade problems.

The Player's Contribution to Accomplishing Equivalence Exchanges Under Thematic Divisions of Labor

As described in the case studies, TON and VER's thematically organized interactions show a further subtlety in how payments can be accomplished in mixed dyads. When TON, the more competent child, was the storekeeper, she structured and accomplished the trade and change goals on her own (with no assistance from VER). In contrast, when VER assumed the role of storekeeper, she relied on TON to help her accomplish the more complex emergent goals of the Trade problems. Indeed, regardless of her thematic role—storekeeper or customer—TON was constructing and accomplishing more complex arithmetical goals than was VER.

We suspected that when fourth graders played the role of customer, they (like TON) would contribute more to the construction and accomplishment of goals than would third graders when they were in the role of customer. To test this, we used a 5-point scale to code the extent to which the player (in this case, the customer) accomplished the emergent trade problem when there was thematic division of labor. Figure 9.7 contains the resulting distribution of mean player contribution scores for the Trade problems when there was thematic division of labor for third graders playing fourth graders

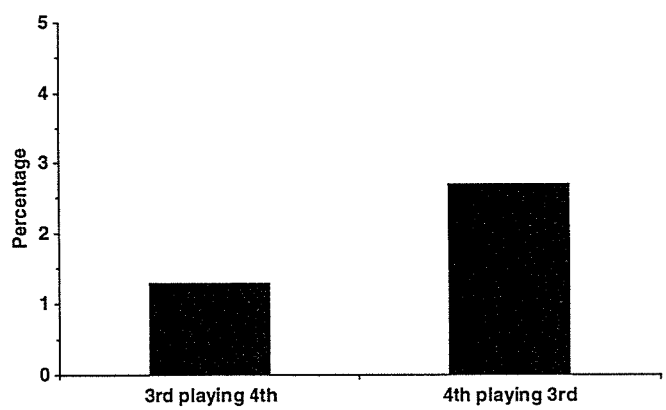

FIG. 9.7. Mean player contribution scores for change problems solved with thematic division of labor.

and for fourth graders playing third graders. Consistent with our expectation, we found that when the turn was the third graders', they contributed little to framing and accomplishing the goals of making change. In contrast, when the turn was the fourth graders', they participated significantly in the construction and accomplishment of the change-making goals.

Summary of Aggregated Analyses

Our aggregated analyses reveal that the character of children's emergent goals differed by grade level of the player and grade level of the player's partner. Dyads as a unit accomplished the majority of the emergent problems accurately. However, when we shifted our unit of analysis from the dyad to the individual, we found that the high accuracy levels for the third graders were due, in part, to the uneven distribution of problem solutions in the course of play: When third graders played fourth graders, they were able to participate in problems that required denominational trades of doubloons without forming goals to produce equivalence trades because of the assistance provided by their partners.

CONCLUDING REMARKS

This chapter made use of case study and aggregated coding methods, both of which focused on emergent goals in play. We see each as compatible with our basic effort to understand learning environments, but these techniques serve different functions. Our case-based analyses provide a window into the particularity of emergent environments for individuals—particularities like the forms of access one child might afford another to the mathematics used in play. Our aggregated analyses provide us with understanding the extent to which the regularities we observe in any particular case might be more general, linked to characteristics of children's prior understandings or the dynamics of dyadic interaction. The information produced by each is wanting, although together these analytic tacks strengthen and enrich one another.

In closing, we note that the analysis of goals that individuals are constructing and accomplishing in practices is a daunting task. Goals are aspects of activity that are invisible to an observer and complexly related to children's understandings and their socioculturally organized activities. Despite the difficulty the construct presents for analyses, we see the focus on emergent goals as an area of inquiry uniquely suited to an analysis of learning environments. Indeed, emergent goals are a pivotal analytic unit because they provide a common ground for the analysis of the constructive, form-building character of children's activities with the analysis of the activity structures, social interactions, and artifact use that are central to understanding cognition in collective practices.

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