Opportunities to Learn Fractions in Elementary Mathematics Classrooms

Maryl Gearhart and Geoffrey B. Saxe, University of California, Berkeley
Michael Seltzer, Jonah Schlackman, Cynthia Carter Ching, and Na’ilah Nasir,
University of California, Los Angeles
Randy Fall, Azusa Pacific University
Tom Bennett, California State University, San Marcos
Steven Rhine, Willamette University
Tine F. Sloan, Santa Barbara, CA

In this study we addressed 2 questions: (a) How can we document opportunities to learn aligned with the NCTM Standards? (b) How can we support elementary teachers’ efforts to provide such opportunities? We conducted a study of the effect of curriculum (problem solving vs. skills) and professional development (subject-matter focused vs. collegial support) on practices and learning. From analyses of videotapes and field notes, we created 3 scales for estimating students’ opportunities to learn. Analyses of fractions instruction in 21 elementary classrooms provided evidence of the technical quality of the indicators and indicated that support for teachers’ knowledge may be required for a problem-solving curriculum to be beneficial.

Key Words: Assessment; Curriculum; Elementary, K-8; Fractions; Professional development; Reform in mathematics education; Teaching practice

In the National Council of Teachers of Mathematics (NCTM) Standards (1989, 1991, 1995), mathematics is represented as a discipline of conceptual inquiry, and mathematics learning is viewed as both a conceptual and a collaborative endeavor (see also California Department of Education, 1992; National Research Council, 1989, 1990). Teachers and students are encouraged to work together to solve problems, analyze conceptual issues, investigate empirical and logical questions, and examine relationships among mathematical representations and their underlying meanings (Ball, 1993; Corwin, 1993; Lampert, 1991). This vision of classroom prac-

The work reported herein was partially supported by National Science Foundation Grant MDR 9154512. The findings and opinions expressed in this report do not reflect the position or policies of the National Science Foundation.

We thank the teachers and their students who participated in our study. Jamal Abedi, Debra Castelan, Thuy Van Le, and Corinne Martinez made contributions to this research. Magdalene Lampert, Martin Simon, and four anonymous reviewers of a prior draft provided insightful comments on our methods and findings.

Correspondence may be sent to the first author.
tice is challenging many teachers, and recent studies indicate that teachers may transform recommended practices in ways that alter the intended functions (Cohen, 1990; Grant, Peterson, & Shoigreen-Downer, 1996; Heaton, 1992; Prawat, 1992a, 1992b; Putnam, 1992; Stein, Grover, & Henningsen, 1996). Effective implementation of a conceptual, problem-solving approach to mathematics instruction appears to require that teachers have deep understandings of both mathematics and the ways that students interpret mathematical problems and build knowledge (Stipek, Gearhart, & Denham, 1997).

Because misalignments between the NCTM Standards’ vision and implementation can result in lost opportunities for student learning, it is important that researchers gather evidence of the contexts and factors that can support effective practices in the classroom. In this study, we address two questions regarding opportunities to learn in elementary mathematics classrooms. The first concerns methods of analysis—How can we document the extent to which students’ opportunities to learn are aligned with the principles contained in documents like the NCTM Standards? The second concerns the roles of curriculum and professional support—How can we best support teachers’ efforts to implement recommended practices in ways that provide students opportunities to learn in ways that are aligned with these Standards documents?

Study Background and Design

Our study was conducted in the state of California following the state’s adoption of the 1992 California Mathematics Framework and prior to the state’s subsequent adoption of textbooks. The state’s interim strategy was the promotion of curriculum units designed to supplement or replace chapters from traditional texts, and our study was concerned with the implementation of two of these units for the upper elementary grades, Seeing Fractions (Corwin, Russell, & Tierney, 1990) and My Travels With Gulliver (Kleiman & Bjork, 1991). These units were designed to support students’ involvement with mathematical problem solving and enhance their conceptual understandings; using these resources, teachers can provide students with multiple models for understanding mathematics in key domains, pose nonroutine and open-ended problems, engage students with multiple representations, and encourage group discussion and problem solving. Seeing Fractions, the problem-solving curriculum unit that is the focus of the work presented in this article, was designed to provide students opportunities to engage with and reflect upon mathematical relationships represented in graphical form. The unit contained a set of five modules; each module contained problems designed to engage students with part-whole relations but in the context of different models of fractions—area models (partitioning squares and circles), fair-sharing models (e.g., constructing fair shares of sets of square brownies or circular cookies; adding shares), and linear models (e.g., comparing fractions strips). The authors of Seeing Fractions designed the unit with the assumption that children’s developing capacities to construct images of part-whole and part-part relationships help them to conceptualize and construct fraction values and to operate on them.
We began our research with a twofold assumption: The activities in these curriculum units have the potential to provide students with opportunities to build conceptual understanding, but teachers need considerable support if they are to implement these problem-solving materials as they were designed (cf. Ball & Cohen, 1996). Our study was designed to help us understand the role of curriculum (problem solving vs. skills) and the kinds of professional support that may enhance students’ opportunities to learn from problem-solving curricula.

We made observations of fractions instruction in elementary classrooms and collected evidence of student learning preinstruction and postinstruction. All teachers were volunteers, willing to allow their practices to be documented and to budget time for participation in the project. In two groups of classrooms, teachers used Seeing Fractions and My Travels With Gulliver; in the third group of classrooms, teachers taught from texts in which skills were emphasized in chapters on fractions, measurement, and scale. All teachers in the first two groups had prior experience with Seeing Fractions and My Travels With Gulliver. These teachers were provided one of two contrasting programs of professional development—Integrating Mathematics Assessment or Collegial Support (both of which are described below). Teachers in the third group, called the Traditional group, were chosen for their expressed commitment to textbooks emphasizing fractions skills. Below we describe the two professional development programs; further details on the selection procedures and criteria are contained in the Methods section. Our comparative design enabled us to investigate how teachers’ choices of curriculum and their opportunities for professional support may lead to different opportunities for upper elementary students to learn fractions. The study design is summarized in Table 1.

<table>
<thead>
<tr>
<th>Group name</th>
<th>Number of participating classrooms</th>
<th>Curriculum</th>
<th>Staff development</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrating Mathematics Assessment</td>
<td>9 9</td>
<td>Problem-solving</td>
<td>Knowledge and assessment</td>
</tr>
<tr>
<td>Support</td>
<td>8 7</td>
<td>Problem-solving</td>
<td>Collegial support only</td>
</tr>
<tr>
<td>Traditional</td>
<td>7 5</td>
<td>Skills</td>
<td>None</td>
</tr>
</tbody>
</table>

*Note. Because of missing data, 3 classrooms were excluded from the analyses reported in this article.*

**Professional Development Programs**

To investigate how different forms of professional support may be linked to students’ opportunities to learn in classrooms, we designed, implemented, and investigated two contrasting professional development programs, each a full year in length, each linked to Seeing Fractions and My Travels With Gulliver, and each aligned with
certain tenets of mathematics education reform. In one program, entitled Integrating Mathematics Assessment (IMA), we provided teachers the opportunities to build their knowledge of mathematics (fractions, measurement, and scale), their knowledge of students’ understandings of concepts and problem-solving strategies in the same domains, and their expertise with assessment. In the second program (Collegial Support), teachers were given the opportunity to build a professional community of like-minded colleagues; teachers met regularly to discuss the challenges of implementing the two curriculum units.

*Integrated Mathematics Assessment Program.* The IMA program was an approach to professional development that incorporated critical components promoted by other research and development efforts in the early 1990s. Findings regarding the effect of professional development on teachers’ frameworks and classroom practices indicated that four elements may be critical to the support of effective instruction: (a) Teachers need a deep understanding of the mathematics they teach—concepts, principles, representations, practices, applications (Ball, 1990a, 1990b, 1993; Brown & Borko, 1992; Leinhardt & Smith, 1985; Post, Harel, Behr, & Lesh, 1991; Shulman, 1987; Thompson, 1992); (b) teachers need a deep understanding of the ways that children learn mathematics; (c) they need to support pedagogies that elicit and build upon students’ thinking (Carpenter & Fennema, 1991; Cobb et al., 1991; Fennema & Franke, 1992; Peterson, 1988); and (d) teachers need to engage in analytic reflection on their practice (Little, 1993; Loucks-Horsley, 1994; Maher, 1988; Richardson, 1990; Schifter & Simon, 1992).

Guided by this analysis, we designed a program that was deeply linked to the mathematics content of *Seeing Fractions* and *My Travels With Gulliver*. The IMA program consisted of cycles of activities, each cycle linked to one model of fractions (area, sharing, or linear) or one conceptual issue in measurement and scale (qualitative vs. quantitative models). For each model or conceptual issue, we designed an integrated series of workshop activities in the following order: Teachers’ Mathematics, Children’s Mathematics, Children’s Motivation, and Implementation of Integrated Mathematics Assessment. The entire IMA program consisted of five cycles through this integrated sequence: three cycles addressing lessons in *Seeing Fractions* and two cycles addressing lessons in *My Travels With Gulliver*. We began with a 5-day summer institute, followed by 13 meetings—1 held approximately every 2 weeks during the year (12 evening meetings and 1 full Saturday meeting). Because this report is concerned just with opportunities to learn and student outcomes in the domain of fractions, we will not provide further information on the measurement and scale components of the IMA program or the Children’s Motivation component (see Stipek et al., 1998).

Teachers’ Mathematics supported teachers’ construction of more sophisticated understandings of fractions. Linked to “big ideas” regarding fractions concepts and strategies for solving *Seeing Fractions* problems, each activity gave teachers opportunities to participate as learners in practices reflective of documents like the *Standards* (NCTM, 1989, 1991, 1995). At any given session, teachers might work
independently to solve an open-ended problem, developing strategies and then analyzing differences among their methods in small-group discussions; or teachers might work in collaborative groups and then consider how their separate contributions to group problem-solving efforts benefited their learning and the quality of their solutions. Instructors functioned as investigators as well as facilitators, observing the teacher-learners while they worked, querying their approaches, and guiding discussions that brought out key mathematical issues that had emerged. Each Teachers’ Mathematics activity was a more complex mathematical investigation than its source in Seeing Fractions. In one fair-sharing activity, for example, IMA teachers were asked to play the role of a pizza store manager and propose strategies for distributing leftover pizza to the homeless each evening; teachers worked in pairs to partition sets of partial pizzas—each pizza either 3/4 of a circle or 2/3 of a rectangle—into fair shares. The variation among teachers’ methods and solutions enabled the facilitator to engage the teachers in reflection on part-whole relations (What is the whole in these problems?), part-part relations (fraction equivalents), and relationships among different representations of fractions. The source of this investigation was a Seeing Fractions lesson in which elementary students partition sets of cookies (circles) or brownies (squares) into fair shares. At the conclusion of each activity, teachers were invited to step back into their roles as teachers and to reflect on practices they had just participated in as learners.

The Children’s Mathematics component was designed to enhance teachers’ interest in assessment of student thinking and their knowledge of children’s mathematical understandings and strategies. Each Children’s Mathematics activity was linked to a Teachers’ Mathematics activity (which in turn was linked to a Seeing Fractions lesson). We presented either videotape snippets of children engaged in solving fractions problems or examples of children’s written work (cf. Carpenter & Fennema, 1991; Cobb et al., 1991); the videotapes and work samples were drawn from pilot classrooms or from individual interviews with children. Our sessions addressed important issues in children’s developing understandings of fractions concepts. One theme concerned the ways that children use their understandings of whole numbers (both representations and operations on the quantities represented) and correspondence relations when attempting to solve fractions problems. For example, a child may divide a quantity represented as a circle or square into four unequal parts and give each of four people “one of these.” Another theme concerned the challenges children face in their efforts to coordinate the meanings of diverse forms of representation (e.g., a drawing vs. a numeric representation of a fraction value). For example, a child who is asked to partition a set of 12 cookies into fair shares for eight people might produce one solution with pictures (“one whole cookie and one of these” [a half]) and another solution with numbers (“8 into 12 is 1 R4”); each representational form affords the child certain interpretations and constrains other interpretations. We engaged teachers in quests to understand children’s efforts to solve and explain mathematical problems involving fractions and to utilize, interpret, and relate different mathematical representations of fractions. Over the course of the year, we shared with teachers the general pattern of children’s developing understandings
of fractions, from whole number approaches to the quantification of part-whole relations. We did not provide a sequenced model of development, unlike the approach taken by the groundbreaking Cognitively Guided Instruction (CGI) program that focuses on early arithmetical strategies in the lower elementary grades (Carpenter & Fennema, 1991); the complexity of fractions and the diversity among Seeing Fractions problems in content and in type made it difficult to provide teachers with sequenced methods of developmental assessment (Lampert, 1991).

The goal of the Implementation component was to enhance teachers’ competence with integrated assessment methods built upon students’ thinking. Activities were always linked closely to the content of the prior Teachers’ Mathematics and Children’s Mathematics sessions. We focused on a range of practices linked to specific Seeing Fractions lessons: whole-class discussions (e.g., how to interpret and integrate “wrong” answers); observation, inquiry, and guidance during student activities (e.g., how to focus the purpose of an observation); assessment of students’ written work (e.g., sample rubrics); peer problem-posing and peer assessment; and portfolio assessment. Teachers analyzed these practices, role-played student and teacher roles, piloted assessment tools, and shared assessments of their own design.

The Collegial Support Program. The Collegial Support program represented an approach to professional support that was promoted in the Greater Los Angeles area at the time of our study. The goal of such programs was to provide teachers opportunities to reflect on their practices with a community of practitioners engaged in similar efforts (Little, 1993; Loucks-Horsley, 1994; Maher, 1988; Richardson, 1990; Schifter & Simon, 1992). Thus the Support program, like the IMA program, provided teachers a context in which to work with other teachers implementing Seeing Fractions and My Travels With Gulliver. However, unlike in IMA, we did not offer sessions designed to enhance the teachers’ knowledge of mathematics, children’s mathematics, or integrated assessment. Instead, Support teachers generated their own agendas for their meetings.

Support teachers met nine times; they began their work on each curriculum unit with a full-day session followed by monthly evening meetings. Topics were suggested by the teachers; the facilitator’s role was to ensure that the same lessons in Seeing Fractions and My Travels With Gulliver that were a focus of IMA sessions were also a focus of Support sessions. Beyond that role, the facilitator supported the teachers’ own agendas by helping the teachers to stay on topic and by sending reminders about the next topic between meetings. Teachers discussed particular practices at some meetings—instructional methods appropriate for specific lessons, the role of manipulatives, assessment methods such as portfolios and open-ended tasks, and homework. At other meetings teachers raised issues about the curriculum units—for example, concerns that there was no one correct answer for many problems, conflicts between the curriculum and what was tested by the teachers’ school districts, and concerns about reducing attention to skills. Each month teachers brought relevant curriculum materials and students’ work to share. Sometimes teachers shared approaches that they felt were successful, and colleagues consid-
ered whether those methods were applicable in their contexts. At other times, teachers either shared methods that were not successful or shared dilemmas they were experiencing and solicited guidance.

*Opportunities to Learn: Developing Measures of Alignment With the Standards*

We examined existing techniques for the analysis of classroom practices in elementary mathematics and found that no existing method provided us with "off-the-shelf" indicators that fit our needs. Our goal was to produce indicators that were reflective of core principles in the *Standards* (NCTM, 1989, 1991, 1995), appropriate to our research questions, and sensitive to the ways classroom practices emerged over the year of our study. We targeted three recommended functions of classroom practices—the degree to which practices elicit and build upon student thinking, the extent to which conceptual issues are addressed in treatments of problem solving, and the extent of opportunity to utilize and interpret representations in ways that help students build understandings of underlying mathematical concepts. We introduce our indicators after we review existing methods.

**Existing measures.** Some observational techniques are designed to capture the existence or frequency of particular actions, instructional functions, or discourse types. Categories have included instructional functions (the conduct of review, checking of homework, time spent on development of the lesson, use of demonstrations), discourse characteristics (kinds of academic questions asked, number of words spoken by teachers vs. by students, initiator of questions), and cognitive complexity (e.g., high- vs. low-level responses) (Ehmeier & Good, 1979; Good & Grouws, 1979; Hiebert & Wearne, 1993; Peterson & Fennema, 1986). We considered the advantages of frequency analyses; we recognized that the specificity of observable events affords measurable elements of practice. However, such approaches could not help us capture the degree to which instructional functions like "building on student thinking" or "integration of conceptual issues with problem solving" emerged in a lesson.

Rating scales appeared more promising, despite their regrettable distance from the actions and discourse that provide evidence for raters' judgments. Use of rating scales enables the analyst to capture dimensions of classroom practice that may be complexly related to specific actions and talk. For example, observers in a study by Good, Mason, and Grouws (1987) rated teachers' actions for the meaningfulness of presentation, degree of emphasis on higher order thinking, and use of manipulatives; observers also rated students' behavior for time on task, evidence of higher cognitive student behavior, and use of manipulatives. The constructs of "meaningfulness of presentation" and "higher order thinking" in this study illustrate the kinds of dimensions of effective practice that can be captured by observers' judgments.

Although the particular dimensions developed by these researchers were not directly helpful to us, we felt that our functional analysis of opportunity-to-learn did require construction of rating dimensions. Using a dimensional framework would allow us to capture various relations between specific forms of practice and the func-
tions served. Recently other researchers developing tools for similar projects have designed tools quite similar to our own. The five-point Classroom Instruction Scales are observers’ ratings of the extent to which four standards of current reforms are realized: higher order thinking, deep knowledge, substantive conversation, and connections to the world beyond the classroom (Newmann, Secada, & Wehlage, 1995). The Inquiry-Based Observation tool, designed to capture the inquiry character of science instruction, contains dimensions of student role (e.g., look for correct answer vs. accept or revise hypotheses on the basis of evidence), teacher role (e.g., source of knowledge vs. facilitator), classroom activities (e.g., algorithms vs. heuristics), and overall emphasis (abstract vs. connected to real world) (Young, Brett, Squires, & Lemire, 1995). The four-level Cognitively Guided Instruction scale for teachers’ instruction provides an index of the alignment of primary level mathematics instruction with the tenets of Cognitively Guided Instruction (e.g., engagement of children in problem solving, elicitiation of children’s thinking, and adaptation of instruction to children’s knowledge) (Fennema et al., 1996). Although these measures of reform implementation are similar in purpose, the scales differ somewhat in underlying assumptions about the processes of implementation and change. The scales of Newmann, Secada, and Wehlage and of Young et al. capture a continuum ranging from traditional to reform practice. In contrast, the lower scores of the CGI scale and our scales do not represent practices that are necessarily “traditional” in character; the ratings are designed to reflect the degree to which a dimension has been implemented, and lower ratings may represent a range of practices.

Our opportunity-to-learn scales. We developed indicators that reflected our interpretation of the goals of the NCTM Standards (1989, 1991) and related documents as well as the research questions specific to our study. These indicators were (a) the degree to which classroom practices elicit and build upon students’ thinking, (b) the extent to which conceptual issues are addressed in treatments of problem solving, and (c) the extent of students’ opportunities to utilize and interpret numeric representations in ways that may help them build understandings of mathematical concepts. Because our data consisted of observations of whole-class lessons, we operationalized the indicators to fit the data, as follows.

The first indicator (Integrated Assessment) represented a central goal of the IMA professional development program as well as a goal of the NCTM Standards documents. We defined integrated assessment as students’ opportunities to participate in classroom discussions built upon their mathematical thinking. Coders were instructed to attend to two practices in particular—teachers’ questioning and public problem solving—and the ways that teachers did or did not use them to elicit and address students’ mathematical understandings.

The second indicator (Conceptual Issues) represented the investment of the Standards (NCTM, 1989, 1991) in enhancing students’ conceptual understanding of mathematical problem solving; children’s conceptual understanding was a core theme in the Children’s Mathematics sessions of the IMA program. In our scheme, we defined conceptual issues as students’ opportunities to consider whether and how
methods for solving problems are linked to fractions concepts—part-whole relations, part-part relations, and equivalence relations. At the lowest levels of our scale, problems are solved with procedures imposed by the teacher (Procedural treatment of fractions concepts and problem solving) or with student-constructed methods that are not analyzed during the whole-class lesson (Discovery treatment of fractions concepts and problem solving). At progressively higher levels, the lesson is increasingly likely to provide opportunities for conceptual analysis of problem-solving methods.

The third indicator (Numerics) represented the Standards’ (NCTM, 1989) emphasis on students’ understandings of multiple forms of mathematical representation. In the IMA program we asked teachers to consider how children’s uses of representations reveal their conceptual understandings of fractions and how different uses of mathematical representations provide students opportunities to reflect on the concepts of fractions. We developed scales for measuring opportunity-to-learn from Numerics and opportunity-to-learn from Graphics, but rater agreement for the Graphics scales was unfortunately unacceptable. When coding Numerics, coders were instructed to attend to the functions of numeric representations in conveying core principles (part-whole relations, equivalence relations).

Our Investigation of Opportunities to Learn

In this article we present findings regarding the development and validation of our three measures of opportunity-to-learn as well as findings regarding relations among curriculum use (problem-solving units vs. textbooks), forms of professional support (IMA vs. Support), and opportunity-to-learn. We expected to document the following pattern of results: (a) Because curriculum units like Seeing Fractions are designed to support class discussions of the conceptual issues underlying strategies for solving problems, we expected to find that the use of Seeing Fractions in both IMA and Support classrooms would be associated with higher ratings on each of our measures of opportunity-to-learn than ratings in Traditional classrooms; (b) further, on the basis of findings in prior studies that indicate that effective implementation of problem-solving curricula requires support for teachers’ knowledge of mathematics and children’s mathematics, we expected that the IMA versus Traditional comparisons would provide the most marked contrasts.

Our data consisted of videotapes and field-note records of classroom observations. Our choices of these data sets were matters of both design and feasibility. Longitudinal collection of data in classrooms spread across the far reaches of the Los Angeles basin was an organizational and a financial challenge, and we had to optimize resources. Videotaping one key fractions lesson over 1 to 2 days provided us relatively raw data and thus the capacity to identify unexpected patterns in classroom practices. Weekly field notes during fractions instruction afforded opportunity for commentary on patterns of implementation over time.
METHOD

Teacher Selection

Volunteers were solicited through mailings to upper elementary teachers within a 40-mile radius of the University of California at Los Angeles. Two letters were distributed: In one mailing we requested applications from teachers engaged with *Seeing Fractions* and *My Travels With Gulliver*; in the second we requested applications from teachers committed to teaching with skills-based textbooks. Both groups were informed that the study would contribute to our understandings of the roles of curriculum in children’s understandings of fractions, measurement, and scale. Applicants were asked to complete a prescreening questionnaire regarding curriculum use, years of experience teaching, degrees and certificates, participation in professional development workshops in mathematics education, grade level(s) taught and currently teaching, student characteristics at their schools, and availability for participation in professional development. Teachers who responded were interviewed to confirm and clarify their responses.

We selected from the respondent pool teachers who (a) were willing to commit to participation in the project for the year and (b) had a history of using texts or the two state-adopted problem-solving units—*Seeing Fractions* and *My Travels With Gulliver*. We assigned to the Traditional (TRAD) group those teachers who had used and would be continuing to use skills-based texts. None of the TRAD teachers had been trained in or had taught either of the two replacement units. We used a stratified random assignment procedure to assign the Integrating Mathematics Assessment (IMA) and Collegial Support (SUPP) teachers. The sample of volunteers who met our curriculum criteria varied on characteristics that were plausibly related to instruction (e.g., prior professional development experience linked to recent reforms, number of years teaching). A simple random assignment procedure was inappropriate with our small sample because by chance the groups might be unbalanced with respect to these characteristics. We describe below the resulting group characteristics following the stratified random assignment of IMA and SUPP teachers.

*Years of experience.* The means and ranges of number of years of teaching experience for the three groups were IMA 16.7 years (range 1–26 years, *n* = 9), SUPP 14.8 years (range 5–22 years, *n* = 7), TRAD 22.8 years (range 4–30 years, *n* = 5). (These statistics represent the teachers whose data are reported in this article.)

*Experience with the problem-solving units.* Almost every teacher in the IMA and SUPP groups had been (a) trained in both the fractions and the measurement/scale units and (b) had previously taught each unit. There were three exceptions: One of the IMA teachers was not trained in the fractions unit (though she had taught it), and two of the IMA teachers had not taught the measurement/scale unit (though they had participated in training).

*Additional professional development.* IMA and SUPP teachers were matched for the extent of their participation in recent mathematics reform workshops. We cre-
ated a scale from 0 to 2 for "additional participation in professional development activities" by assigning one point for training in any other mathematics curriculum replacement unit and one point for any other professional development in mathematics education; the mean for IMA teachers was 1.3 (range 0–2) and for SUPP teachers 1.1 (range 0–2). The mean for TRAD teachers was 0.6 (range 0–1), lower than for the other two groups, as we expected, given their expressed commitment to teaching skills. At the time of our study, there existed few professional development opportunities for teachers committed to a skills approach to mathematics teaching.

Student characteristics. Decisions about assignments of teachers to groups were made, to the extent feasible, on the basis of school data on ethnicity and language use from the prior year and on the grade level that each teacher anticipated teaching in the fall. Because we were initiating our professional development programs in the summer, the timing precluded assigning teachers to groups according to the characteristics of the students who were eventually assigned to their classrooms.

The students participating in the study represented the diversity of Greater Los Angeles. In the entire sample, 64% of the students were Latino, 14% were White, 8% were African American, and 7% were Asian. Table 2 contains background data for the students in each study group—grade level, English fluency, knowledge of fractions. The measure of English fluency was a rating on a four-level scale of fluency and capacity to participate in English-only instruction; our ratings were derived from the school's categorical assignment as well as from teachers' judgments. The measure of students' fractions knowledge was total score on a fractions paper-and-pencil preassessment (Saxe & Gearhart, 1998). ANOVAs and Duncan post hoc comparisons indicated group differences for students' pretest performance ($F(2, 18) = 3.5150, p = .0514$; IMA different from SUPP). There were no group differences in English proficiency.

<table>
<thead>
<tr>
<th>Student characteristic</th>
<th>IMA ($n = 9$)</th>
<th>Support ($n = 7$)</th>
<th>Traditional ($n = 5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>5.33</td>
<td>4.83</td>
<td>4.74</td>
</tr>
<tr>
<td>$SD$</td>
<td>0.71</td>
<td>0.84</td>
<td>0.73</td>
</tr>
<tr>
<td>English proficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>2.94</td>
<td>2.78</td>
<td>2.65</td>
</tr>
<tr>
<td>$SD$</td>
<td>0.09</td>
<td>0.22</td>
<td>0.39</td>
</tr>
<tr>
<td>Pretest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>7.42</td>
<td>3.97</td>
<td>4.57</td>
</tr>
<tr>
<td>$SD$</td>
<td>2.82</td>
<td>1.82</td>
<td>3.67</td>
</tr>
</tbody>
</table>

*Note. These student data represent the classrooms participating in the analyses reported in this article. For each student characteristic, a classroom mean was computed, and the group mean is the mean of the classroom means.*
Data Collection and Specification

Videotapes were made of the same lesson on addition of fractions (Seeing Fractions, Module 3, Lesson 3) in the IMA and SUPP classrooms; videotapes in the TRAD classrooms captured each teacher’s opening lesson on addition of fractions. Field notes were collected in IMA and SUPP classrooms once a week throughout the teaching of area and sharing models of fractions (Seeing Fractions, Modules 1 and 3); in the TRAD classrooms, field notes captured typical instruction during each teacher’s fractions unit. Field notes were organized into three columns: on the left, uninterpreted observations of discourse and actions; on the right, the observer’s comments; in the middle, summary labels that identified the mathematics problem and phase of activity. Drawings and teacher handouts were appended and indexed to the notes. All field notes were submitted to the first author as drafts for review and revision, with the exception of one set per classroom prepared by the first author.

We chose to develop our scheme for whole-class episodes because the quality and completeness of videotapes and field notes during periods of student activity (individual and cooperative work) were uneven. A whole-class episode was defined as (a) teacher-supervised activity and interaction, (b) the function of which was either to prepare students for independent or cooperative work on similar problems (introduction) or to discuss work that students had completed independently or cooperatively (closing). (The boundaries between whole-class and student activity were clear for most videotapes and field notes. Unclear cases were those in which the teacher supervised students’ work closely, organizing the event as a repeating pattern of partial solution and discussion, partial solution and discussion, etc.; in such cases, coders were instructed to attend only to the segments of whole-class discussion.) Most videotapes or field notes contained both an introduction and a closing, and both were used as evidence for coding decisions, although codes were applied only once to the entire tape or set of field notes.

Teachers’, students’, and observers’ names on videotapes and field notes were replaced with random identification numbers that did not reveal study-group membership.

Coding Scheme

The material used for scheme development was drawn from our videotapes and field notes. Every effort was made to remain “blind” with respect to teacher identity and group membership; however, because we differed in our familiarity with both, during training we engaged in extended discussion of the evidence for our judgments to help us identify possible biases.

The videotape and field-note schemes were designed to provide evidence of the same dimensions of opportunity-to-learn, and thus the schemes paralleled one another. Scales were established as 3 or 4 points on the basis of the capacity of the scale to support rater agreement during scheme development. Tables 3, 4, and 5 contain the scales that demonstrated acceptable levels of rater agreement and are therefore reported in this article; the full scheme is available from the first author. Integrated Assessment captures stu-
students' opportunities to participate in classroom discussions built upon students’ mathematical thinking. Coders were instructed to attend to two practices in particular—teacher questioning and public problem solving—and the ways that these did or did not elicit and address students’ mathematical understandings. Conceptual Issues captures students’ opportunities to engage with conceptual issues underlying problem solving: core concepts include part-whole relations, part-part relations, and equivalence relations. The Numerics scales capture students’ opportunities to represent or interpret fractions or operations on fractions with numeric representations. When coding Numerics, coders were instructed to attend to the functions of numeric representations in conveying core principles (part-whole relations, equivalence relations) and strategies for solving problems involving fractions (combining fractional quantities).

<table>
<thead>
<tr>
<th>Rating</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Ongoing effort to ascertain students’ understandings</td>
</tr>
<tr>
<td></td>
<td>Students in this classroom would expect that the teacher and students are invested in understanding their interpretations of the mathematics. Questions provide access to students’ understandings of major components of the problem: The teacher or students ask students to explain their work or their reasoning; when students explain, someone (usually the teacher) extends the responses and explores what the student understood. The teacher may (a) make an interpretation of the response, (b) ask students to make an interpretation of the response, (c) encourage comparison with another student’s strategy/reasoning, etc. The teacher and students show evidence of valuing ways that students approach and understand problems.</td>
</tr>
<tr>
<td>3</td>
<td>Some effort to ascertain students’ understandings</td>
</tr>
<tr>
<td></td>
<td>Students in this classroom would expect to be asked some questions requiring a more extended response or public solution of a problem (or a major component of a problem) but would not expect that the teacher or any student would be consistently invested in understanding their interpretations of the mathematics, nor would they expect that they could contribute an interpretation of another student’s work. Questions provide some or inconsistent access to students’ understandings. If the teacher asks students to explain their reasoning or to solve and discuss a problem publicly, he or she does not extend the response; the purpose of any follow-up question appears to be to check on the student’s use of a required procedure.</td>
</tr>
<tr>
<td>2</td>
<td>Limited effort to ascertain students’ understandings</td>
</tr>
<tr>
<td></td>
<td>Students in this classroom would not expect that the teacher or any student would be invested in understanding their interpretations of the mathematics. Questions provide very limited access to students’ understandings; the purpose of any question seems to be to ensure student attention or the step-by-step completion of a solution to a model problem. When asked a question, students are typically required to choose a possible response or to respond “yes” or “no”; content is typically focused on specific solution steps or the solutions. (Solutions may be oral, numeric representations, or graphic representations.)</td>
</tr>
<tr>
<td>1</td>
<td>Very little effort to ascertain students’ understandings</td>
</tr>
<tr>
<td></td>
<td>Questions are rarely asked. Neither the teacher nor other students ask students to explain their reasoning. Students do not work through a problem publicly in ways that could reveal their understandings or strategies.</td>
</tr>
</tbody>
</table>

Coding Procedures

Five coders were trained on a core set of videotapes and field notes, and most videotapes and field notes were then coded by more than one coder. Coders were permitted to assign a plus or minus to a scale point. A plus or minus was given the value
Table 4
Rating Scale for Conceptual Issues Integrated With Problem-Solving Procedures

<table>
<thead>
<tr>
<th>Rating</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Extensive integration</td>
</tr>
<tr>
<td></td>
<td>4: A fundamental goal of the discussion appears to be to engage students in formalizing procedures the students have devised or engaging students in conceptual analysis of conventional procedures. The discussion may focus on the ways procedures address and can reveal part-whole relationships or operations on fractional quantities. Discussions generally entail consideration of conceptual relations among possible representations (graphic, numeric, linguistic) for fractional quantities or operations on fractional quantities.</td>
</tr>
<tr>
<td>3</td>
<td>Some/occasional integration</td>
</tr>
<tr>
<td></td>
<td>3P (Procedural): During the discussion, only one procedure is recommended or “right,” and this procedure may be linked to graphics or manipulative patterns in ways that could help students understand fractions in terms of part-whole relations or operations on fractional quantities. Other procedures may be acknowledged (e.g., “6/4 is the same as 1 2/4, and that is good thinking, but this time we are working on a solution expressed as a mixed fraction only”); however, students may be left with uncertainty regarding the conceptual distinctiveness or similarity of different procedures.</td>
</tr>
<tr>
<td></td>
<td>3D (Discovery): The primary goal of the discussion appears to be to collect multiple paths to correct solutions (“so this is another way we can solve this problem”); there is weak and unpredictable analysis of relations among the correct solutions; incorrect solutions are either excluded from discussion or are noted as incorrect but not examined. Thus there is an effort at comparison or analysis of solution paths that addresses part-whole relations or combining fractional quantities; however, students may be left with uncertainty regarding the conceptual distinctiveness or similarity of different solution paths.</td>
</tr>
<tr>
<td>2</td>
<td>Limited integration</td>
</tr>
<tr>
<td></td>
<td>2P (Procedural): The goal of the discussion appears to be to confirm that students follow a required procedure and solve the given problem. Only one procedure is used or considered “right.” (Although the procedure may not be explicitly required, students seem to know what they are “supposed to do.”) There is little analysis that could help students reflect on part-whole relations or on the meaning of the algorithms used for combining fractional quantities.</td>
</tr>
<tr>
<td></td>
<td>2D (Discovery): The goal of the discussion appears to be to collect many ways that students have discovered to solve the problem. Students construct their own methods and share their strategies with the class. However, the discussion does not highlight the relations among strategies and either part-whole or operations on fractional quantities; strategies that result in incorrect solutions are left unexamined.</td>
</tr>
<tr>
<td>1</td>
<td>Not integrated</td>
</tr>
<tr>
<td></td>
<td>1P (Procedural): The teacher prescribes a predefined series of steps for solving the problem that he or she may list on the board for reference; the goal appears to be to teach steps for solving addition-of-fractions problems. In the discussion, there is no analysis of these steps that could help students reflect on part-whole relations or on the meaning of the algorithms used for combining fractional quantities.</td>
</tr>
<tr>
<td></td>
<td>1D (Discovery): The goal of the discussion is to acknowledge that students have discovered many ways to solve the problem. Student sharing is brief, and what students say or show is generally unclear; every method or solution is found interesting.</td>
</tr>
</tbody>
</table>

of ± .33. Thus a 1+ was assigned a value of 1.33 and a 2− was assigned a value of 1.67. (Values could not be less than 1 or greater than the highest whole number value on the scale.)

Videotape coding and field-note coding involved two steps: (a) initial review, identification of whole-class episodes, and annotations of evidence from those episodes bearing on the coding dimensions; (b) coding, with additional records of evidence as necessary. Experienced coders were able to code most videotapes in 3 to 4 hours, field notes in 1 hour.
Table 5
Rating Scale for Opportunity to Gain Understanding of Concepts Linked to Uses of Numeric Representations

<table>
<thead>
<tr>
<th>Rating</th>
<th>Opportunity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Opportunity</td>
<td>In this discussion, uses of numerics are likely to support all students’ understandings of the ways fractional quantities and operations on fractional quantities are represented in numeric representations. Written numerics are used frequently and are complete and legible; written numerics are generally consistent in format. Oral numerics are used frequently, are complete, audible, and generally consistent in format. If variations in written or oral format emerge in the discussion (e.g., orally, “a fourth,” “one fourth,” “a quarter”), the teacher makes note of these variations and discusses them. The locations of the numerator and denominator are identified. Students’ attention is drawn to relationships between oral and written numerics and to the relation of both to part-whole relationships.</td>
</tr>
<tr>
<td>2</td>
<td>Some opportunity</td>
<td>In this discussion, uses of numerics may be understood by students who have prior knowledge of fractions concepts, but they are not likely to be helpful to students who do not have prior knowledge of fractions concepts. Written numerics are used inconsistently or, when used, are sometimes sketchy or difficult to read. Oral numerics may sometimes be inaudible. Variations in written or oral format emerge in the discussion (e.g., orally, “a fourth,” “one fourth,” “a quarter”); the teacher may make note of these variations but not discuss them. The locations of the numerator and denominator are identified. The lesson draws students’ attention to relationships between oral and written numerics or to the relationship of both to part-whole relationships, but infrequently or in confusing ways.</td>
</tr>
<tr>
<td>1</td>
<td>Little opportunity</td>
<td>In this discussion, uses of numerics may be confusing to all students. Written numerics are infrequent, sketchy, difficult to read. Oral numerics are sometimes inaudible. When variations in written or oral formats are used, the teacher makes no note of these. The locations of the numerator and denominator may not be identified. The lesson does not draw students’ attention to relationships between oral and written numerics or to the relationship of both to part-whole relationships.</td>
</tr>
</tbody>
</table>

Measuring Students’ Learning

To document children’s understandings of fractions, we used a paper-and-pencil test (Saxe & Gearhart, 1998; Saxe, Gearhart, & Seltzer, in press). Items were constructed to reflect typical fractions problems in upper elementary texts as well as the more open-ended and nonroutine problems that are reflective of problem-solving curricula. The paper-and-pencil test was administered by project staff members to students in participating classrooms both before and after the intervention. Students were offered the option of taking the test in Spanish or English. The duration of the test was about 40 minutes.

On the basis of an item analysis of the fractions test, we created two subscales. The Computation items required students to add and subtract fractions, compute fraction equivalencies, and express values in a pie chart; each of these problems is readily solved with recipe-like application of algorithms. The Problem Solving items required students to construct fractions for unequal parts of wholes, estimate fractional parts of areas, and solve fair-share problems by producing both a graphical and a numeric solution; none of these problems could be easily solved through the mere application of a computational algorithm. Cronbach’s alpha indicated internal consistency for each scale (Problem Solving—.73 [pretest] and .83 [posttest]; Computation—.86 [pretest] and .87 [posttest]). Confirmatory factor analysis pro-
vided strong support for our use of the two scales (Saxe & Gearhart, 1998), and, thus, in the results that follow, we use students’ scores on these scales.

RESULTS

We report (a) findings regarding the technical quality of our measures of opportunity-to-learn as well as (b) findings emerging from our intervention study. The results for technical quality are presented first: rater agreement, data reduction (correlations and principal components analysis), and relations between our measure of opportunity-to-learn and our measure of student learning. We then use our validated measures to analyze the effect of curriculum and professional development on students’ opportunities to learn.

Technical Quality

Rater agreement. We asked first whether raters agreed in their judgments of opportunity-to-learn. For each rating scale, we used ITRS software (Abedi, 1996) to examine evidence of rater agreement using an array of indices, including percentage agreement (exact and within one point), average correlation, Kappa, and Cronbach’s alpha. Indices were computed for videotapes and field notes coded by all raters and for videotapes and field notes coded by each pair of raters. Percentage agreement and correlations are measures of reliability that are easy to understand and interpret. Kappa is a measure of agreement that can be used for any number of raters and for ordinal data. Cronbach’s alpha (with raters as items) is another well-accepted measure of rater agreement.

On the basis of these results, we determined that one rater’s codes showed poor consistency with the codes of the other raters, and this rater’s codes were dropped from the analysis. (This individual joined the pool late and coded only a small number of videotapes.) We then examined the pattern across all the statistics computed by the ITRS software to make judgments about which scales to include in the analysis. Table 6 shows results for the scales that we considered to be adequately reliable. Please note that the findings in Table 6 for Cronbach’s alpha and average correlations are limited to cases with data for all the raters; there were only three to seven such cases for each of our scales. Therefore, when making our decisions about which scales to retain, we also examined Cronbach’s alphas and correlations for each possible rater pair; to conserve space, we omitted these findings from the table. The ns for these rater-pair statistics ranged from 6 to 12, and the indicators were positive (.32 to .93 for Cronbach’s alpha and .39 to .87 for correlations). The rater-pair statistics justified our retention of the Video Assessment scale (see Table 6); although Cronbach’s alpha for Video Assessment could not be computed because of negative interitem correlations, rater-pair statistics were acceptable (.45 to .77 for Cronbach’s alpha and .39 to .63 for correlations), particularly in conjunction with a finding of 100% agreement within one scale point for each rater pair.
Table 6
Indices of Coder Agreement for the Scales Retained

<table>
<thead>
<tr>
<th>Scales</th>
<th>Cronbach’s alpha</th>
<th>Average correlation</th>
<th>Percentage agreement</th>
<th>Percentage agreement</th>
<th>Within 1 point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field-note scales</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assessment</td>
<td>(4)</td>
<td>.87</td>
<td>(4) .69</td>
<td>(88) .22</td>
<td>(88) .92</td>
</tr>
<tr>
<td>Conceptual</td>
<td>(7)</td>
<td>.90</td>
<td>(7) .75</td>
<td>(80) .30</td>
<td>(80) .93</td>
</tr>
<tr>
<td>Videotape scales</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assessment</td>
<td>(3)</td>
<td>—.6 c</td>
<td>(3) -.26</td>
<td>(27) .15</td>
<td>(27) 1.00</td>
</tr>
<tr>
<td>Conceptual</td>
<td>(3)</td>
<td>.87</td>
<td>(3) .86</td>
<td>(27) .19</td>
<td>(27) .96</td>
</tr>
<tr>
<td>Numerics written</td>
<td>(3)</td>
<td>.65</td>
<td>(3) .53</td>
<td>(23) .13</td>
<td>(23) 1.00</td>
</tr>
<tr>
<td>Numerics oral</td>
<td>(3)</td>
<td>.83</td>
<td>(3) .87</td>
<td>(23) .30</td>
<td>(23) .96</td>
</tr>
</tbody>
</table>

Note. Decisions were made on the basis of findings for all combinations of rater pairs; these data are available from the first author.

a n represents the number of field notes or videotapes rated by all the raters.
b n represents the number of opportunities to compare the ratings of two raters (e.g., if three raters rated a videotape, there are three possible pairwise comparisons for that videotape).
c Cronbach’s alpha for Videotape Assessment could not be computed because of negative item correlations.

The results for rater reliability necessitated the removal of field-note Numerics (FNUM) ratings from our data set. Elimination of FNUM then necessitated the removal of all data from three classrooms from further analysis (one Support and two Traditional) because, for these three classrooms, we had available only field notes (only FNUM scores) and no videotapes (no VNUMOR or VNUMWR scores).

Data reduction. To eliminate redundancy across our data, we used a series of steps to reduce the data to meaningful scores of opportunity-to-learn for each classroom.

We began by producing scores for each classroom on each of our scales, and we illustrate these procedures for videotape coding. First, for each videotape coded, we averaged ratings on each scale across raters. Second, we averaged the scores for all videotapes collected in a classroom to produce one measure for that classroom on each scale. The four measures constructed in this way from ratings of videotapes (V) were labeled as VASS (video evidence of opportunities for integrated assessment of one’s thinking), VCONC (video evidence of opportunities to engage with a conceptual treatment of problem solving), VNUMWR (video evidence of opportunities to create or interpret written representations of fractions and operations with fractions using numerals and arithmetic symbols), and VNUMOR (video evidence of opportunities to use or interpret oral representations of fractions and operations with fractions using numerals and arithmetic symbols). Analogous field-note measures (F) were labeled as FASS and FCONE. (FNUM ratings had already been eliminated because of weak rater agreement.)

We then used correlations and Cronbach’s alphas to provide initial evidence of relationships among these six measures. Cronbach’s alpha (which assumes unidimensionality) for one set of scores containing \{VNUMWR and VNUMOR\} was
α = .92, indicating high consistency. Cronbach’s alpha for a second set containing \{VASS, VCONC, FASS, and FCONC\} was α = .88, again indicating high consistency.

We used the method of principal components for variable reduction (Stevens, 1992, p. 375).\(^1\) For this analysis, we adopted two criteria for retaining components: (a) retain all components with eigenvalues greater than 1 and (b) retain all components explaining more than 10% of the total variance of the original variables. We used a scree plot to inspect visually how many components should be retained, selecting components above the “elbow” point. The analysis on the six measures—VASS, VCONC, FASS, FCONC, VNUMWR, and VNUMOR—for the 21 classrooms retained two principal components, explaining 83.9% of the total variance of the original variables. The first principal component was weighted more by the variables VASS, VCONC, FASS, and FCONC (Y₁ = .31 * VASS + .26 * VCONC + .28 * FASS + .31 * FCONC - .12 * VNUMWR + .001 * VNUMOR). The second principal component was weighted more heavily by the variables VNUMWR and VNUMOR (Y₂ = -.025 * VASS - .13 * VCONC - .03 * FASS - .16 * FCONC + .51 * VNUMWR + .46 * VNUMOR). Thus the results provided evidence of two underlying dimensions: (a) students’ opportunities to engage with conceptual issues in fractions in whole-class discussions of problem solving that were built on assessment of their thinking and (b) students’ opportunities to use and interpret numeric representations of fraction values and operations on fractions in ways that capture important underlying fractions concepts. We used the two principal component scores in place of the original six variables to describe students’ opportunities to learn. We named the two scores Conceptual Analysis Built Upon Assessment of Student Thinking (or Conceptual/Assessment) and Numerics.

Relations between opportunities to learn and students’ learning. To provide evidence of validity for the principal components scores, we examined relations between these two indices of opportunity-to-learn and our two indices of students’ learning (the Problem Solving scale and the Computation scale). On the one hand, we assumed that if the Conceptual/Assessment and Numerics scores provided valid indicators of opportunities for conceptual learning, then classrooms that receive higher scores should show greater gains from pretest to posttest on the Problem Solving scale of the fractions test (predictive validity). On the other hand, our scores of opportunity-to-learn should not predict greater gains from pretest to posttest on the Computation scale (discriminative validity), because the items contributing to the Computation scale were designed to assess students’ performance on items that could

---

\(^1\) Principle components analysis is a psychometrically sound procedure that is mathematically simpler than principal factor analysis, in which the issue of factor indeterminacy may be troublesome. To determine the underlying “components” of a set of variables, in principal components analysis one partitions the total variance (the sum of the variances of the original variables) by finding linear combinations of the variables that account for the maximum amount of variance: Y₁ = d₁₁X₁ + d₁₂X₂ + ⋯ + d₁pXₚ (where Y₁ is called the first principal component of the original Xₚ variables). One then finds a second linear combination that is uncorrelated with the first component and thus accounts for the next largest amount of variance, beyond the variance explained by the first component. The process is repeated until all components explaining significant amounts of variance are found.
be solved with routine algorithms and mathematics facts, not conceptual understanding of fractions.

We used Hierarchical Linear Modeling (HLM) to analyze relations between our indices of opportunity-to-learn and students’ learning (see Bryk & Raudenbush, 1992). For analyses of nested data (e.g., students nested within different classrooms), HLMs are more appropriate than standard regression techniques because the standard errors obtained via HLMs are adjusted for the degree of dependency (i.e., the degree of intraclass correlation) among observations within classrooms.

In our models, class mean posttest scores for both problem solving and computation problems were modeled as a function of our two measures of opportunity-to-learn, adjusting for differences among classes in pretest performance (see, e.g., Bryk & Raudenbush, 1992, pp. 19, 96–102). Thus, akin to techniques such as multiple regression and ANCOVA, our models yielded estimates of the effects of opportunity-to-learn on posttest performance, holding constant pretest performance and language proficiency. Also akin to the use of covariates in multiple regression and ANCOVA techniques, the inclusion of covariates in our analysis—in particular, pretest performance—provided added power in our analyses.

As predicted, the results showed that opportunity to engage in Conceptual Analysis Built Upon Assessment of Student Thinking predicted Problem Solving posttest performance ($t = 4.09, p < .001$). In terms of practical significance, the results indicated that for each 1-unit increase on the Conceptual/Assessment Opportunity scale (a 4-point scale), we tended to see an increase of approximately 0.87 points on the Problem Solving scale (a 13-point scale). Note that when we omitted the pretest performance and language background covariates from the model, we obtained quite similar results. The resulting estimate of the effect of Conceptual/Assessment Opportunity on Problem Solving performance was slightly larger (1.01 vs. 0.87), and the resulting standard error was, as expected, slightly larger (0.34 vs. 0.21) because of a reduction in power. An important point, however, is that in both cases, the results were highly statistically significant. Thus we found that students in classrooms with higher ratings for opportunity to engage in conceptual discussions built on their understandings of fractions were more likely to gain in their performances on the Problem Solving items.

Also as predicted, there were no significant relationships between either the Conceptual/Assessment or the Numerics Opportunity scores and the Computation posttest scores. The rating scales contributing to the two opportunity scores were not designed to capture students’ opportunities to learn basic skills and thus should not have predicted students’ performance on skills items.

One finding was inconsistent with our predictions—there was no relationship between the Numerics Opportunity score and the classroom Problem Solving posttest score. This finding may reflect the limitations of using a measure of opportunity focused on one form of representation (numerics) when other forms served criti-

---

2 See the Appendix for greater detail on the HLM models.
cal representational functions in classroom practices (e.g., graphics). Unfortunately, as we previously noted, rater agreement on our Graphics scale did not meet our criteria, and, as a consequence, we could not test this relation.

We conducted a second analysis to address a competing explanation of the link between the Conceptual/Assessment Opportunity scale and student performance on the Problem Solving scale. In our sample, the classrooms with higher Conceptual/Assessment scores tended to have students with higher pretest scores; it is possible that students with more knowledge may have progressed at a greater rate than those with less knowledge (cf. Cipielewski & Stanovich, 1992; Stanovich & Cunningham, 1993). If so, opportunity-to-learn may have contributed little or nothing to learning gain. Evidence for this competing explanation would weaken our claims for the validity of our Conceptual/Assessment scale. To determine the merits of this explanation, we correlated students’ pretest Problem Solving scores with the difference between pretest and posttest Problem Solving scores (i.e., Problem Solving gain). We found no evidence that prior knowledge was related to gain: At the classroom level ($n = 21$), correlations between classroom pretest Problem Solving means and mean Problem Solving gain scores were not significant ($r = .14, p = .54$). These findings indicate no relation between prior knowledge and learning gain. Thus students’ prior knowledge is not a viable explanation for the association between opportunity to engage in Conceptual Analysis Built Upon Students’ Thinking and students’ Problem Solving achievement.

**Summary of findings regarding the technical quality of our measures of opportunity-to-learn.** We found ratings of videotapes and field notes to be adequately reliable for most scales. Correlations and principal components analyses of the reliable scores produced two readily interpretable scores: (a) students’ opportunities to engage with conceptual issues in fractions in whole-class discussions built upon their thinking (Conceptual/Assessment Opportunity) and (b) students’ opportunities to engage with numeric representations of fraction values and operations in ways that capture underlying fractions concepts (Numerics Opportunity). We found evidence of the predictive and discriminative validity of the principal components scores. As predicted, the Conceptual/Assessment scale predicted gains in students’ performance on the Problem Solving items; also as predicted, neither the Conceptual/Assessment scale nor the Numerics scale predicted gains in students’ performance on the Computation scale. Inconsistent with our prediction, Numerics did not predict gain in performance on Problem Solving items.

**Group Differences in Opportunity-To-Learn: Roles of Curriculum and Professional Development**

We used our measures to analyze the effects of curriculum and professional development on students’ opportunities to learn, recognizing that evidence for the validity of Numerics was less strong than the evidence for Conceptual/Assessment. We used two univariate analyses to examine whether and how curriculum and professional development contributed to differences in opportunity-to-learn. Univariate
methods were appropriate choices for examining group effects on our principal component scores inasmuch as the principal component scores were constructed to be independent.  

We found differences among the three groups on our two measures of opportunity-to-learn: Conceptual Analysis Built Upon Assessment of Student Thinking \(F(2, 18) = 15.53, p < .0001\) and Numerics \(F(2, 18) = 5.74, p = .0118\). We undertook pairwise comparisons (Tukey HSD) on group means for both principal component scores, and these results are shown in Table 7. For Conceptual/Assessment, there were statistically significant \((\alpha = .05)\) differences between IMA and TRAD and between SUPP and TRAD classrooms. For the Numerics score, there were statistically significant differences between IMA and SUPP and between SUPP and TRAD classrooms.

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Differences Between Groups in Opportunities to Learn: Results of ANOVAs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conceptual/Assessment Opportunity</td>
</tr>
<tr>
<td>Principal component scores</td>
<td></td>
</tr>
<tr>
<td>IMA</td>
<td>0.5730</td>
</tr>
<tr>
<td>SUPP</td>
<td>0.2381</td>
</tr>
<tr>
<td>TRAD</td>
<td>-1.3647</td>
</tr>
<tr>
<td>Group comparisons (Tukey-HSD)</td>
<td></td>
</tr>
<tr>
<td>IMA-SUPP</td>
<td>X</td>
</tr>
<tr>
<td>IMA-TRAD</td>
<td>X</td>
</tr>
<tr>
<td>SUPP-TRAD</td>
<td>X</td>
</tr>
</tbody>
</table>

The results for Conceptual/Assessment were consistent with our prediction that the problem-solving curriculum unit would give students greater opportunities to engage with the conceptual issues of fractions than would textbooks emphasizing skills. Significant IMA-TRAD and SUPP-TRAD comparisons indicated that classrooms using the new curriculum were providing students greater opportunities to engage with conceptual issues during whole-class discussions that were built upon assessment of their thinking. The pattern of group differences was different for Numerics. Compared with TRAD classrooms using textbooks, SUPP classrooms implementing the problem-solving curriculum had relatively lower ratings for providing students opportunities to learn how numeric representations are linked to fractions concepts. IMA ratings on Numerics Opportunity were no different from TRAD ratings and were also significantly higher than SUPP ratings. This pattern

---

3 As shown in Table 2 and as previously discussed, groups differed in students’ prior knowledge of fractions (PRETEST) and in students’ competency in English (LANGBACK). Either of these student variables could plausibly contribute to students’ opportunities to learn. However, no correlation between either of these student measures (aggregated at the class level) with the opportunity-to-learn scores was significant. Thus, with no evidence that students’ prior knowledge of fractions or students’ English fluency was contributing to their opportunities to learn, there was no need to remove the variance contributed by these student indicators.
of findings indicated that a curriculum emphasizing graphic representations is at risk for reducing students’ opportunities to engage productively with numeric representations; however, a professional development program, like IMA, designed to enhance teachers’ understandings of the mathematics can mitigate this risk.

SUMMARY AND IMPLICATIONS

Strategies to support effective implementation of reform curricula and pedagogy are emerging (Loucks-Horsley, 1994; Stipek et al., 1997), but much remains that is not understood about the effects of professional support on classroom practices and students’ learning. With this study we have contributed both new methods of evaluation and new evidence bearing on the crucial question “What works?” in mathematics education at the elementary level. We conclude with a summary of our study and commentary on the implications of our findings.

Summary

Our analyses were designed to accomplish two key and related purposes. One purpose was to validate new measures of opportunity-to-learn for use at the upper elementary level, measures aligned with the tenets of mathematics education reform. The second purpose was to use the validated measures to evaluate the roles of contrasting curricula and contrasting models of professional support in students’ opportunities to learn.

Evidence of validity for the opportunity-to-learn measures. Capturing and quantifying aspects of classroom practice, from the student’s perspective, as opportunity-to-learn is a complex endeavor, and we achieved some degree of success in validating a new approach. We also encountered certain challenges.

The good news was of several types. First, most of the scales that we developed were applied reliably, an important finding if we consider the complexity of the task of rating opportunity-to-learn on the basis of videotapes and sets of field notes. Second, the two scores that emerged following statistical procedures for reducing the data were interpretable and reflected important recommendations contained in the NCTM Standards (1989, 1991, 1995) and related documents. One score captured students’ opportunities to learn from lessons that address conceptual issues underlying problem solving in ways that build on their thinking. Another score captured students’ opportunities to gain understandings of the ways that numeric representations are linked to fractions concepts. Third, one of the scores, Conceptual Analysis Built Upon Assessment of Student Thinking, predicted students’ performance on a measure of problem solving, and it did not predict students’ performance on a measure of computation. Fourth, univariate analyses of both the Conceptual/Assessment and the Numerics opportunity-to-learn scores produced interpretable group comparisons, and these findings can be considered as both evidence of validity for the measures and evidence of the roles of our interventions in opportunity-to-learn. (We will return to discuss these findings shortly.) Collectively,
these findings indicate that we have designed reliable and valid methods of rating videotapes and field notes as evidence of students’ opportunities to learn from discussions, methods that define opportunities in terms of key principles of inquiry mathematics education.

Some of the findings raised questions regarding the constructs underlying our original scales, weaknesses in the specifications of those scales for raters, or both, and further work is needed to resolve the questions raised by these findings. Scores for two of the original scales—Conceptual Analysis of Problem Solving and Integrated Assessment—were highly correlated, a result that may reflect an overlap in coding specifications. We now recognize that application of both schemes required a rater’s attention to the characteristics of the discourse, particularly teachers’ questions and students’ contributions.

In addition, the Numerics opportunity-to-learn score was not related to students’ performance on our Problem Solving scale as we predicted. This finding raises questions about both the Numerics scale and the student-achievement measure. First, it is possible that a different student measure might have produced the predicted relationship. Some of the items in the Problem Solving scale were scored in a way that gave students partial credit for correct graphics if the numerical representations were incorrect, and that provision in the scoring may have reduced the likelihood of a relationship with the Numerics Opportunity score. Second, it is possible that the Numerics Opportunity scale was not adequately capturing what was intended; the scale did not contain explicit examples of uses of numeric representations, and indeed the omission of examples may explain why Numerics scores did not predict either of the student-outcome scores.

Evidence of the roles of curriculum and professional development. The principles underlying recent reforms in mathematics education are complex, and recent studies are demonstrating that the translation of this vision for reform into classroom practice is no easy feat. New curriculum materials and new approaches to professional development have been designed to support teachers in their efforts, but too little is known about the ways variations in either of these factors may enable changes in the classroom. Through our study we contributed new evidence regarding the roles of curriculum and professional development.

Our findings for Conceptual Analysis Built Upon Assessment of Student Thinking showed that use of a curriculum that reflects reform recommendations can have positive effects on students’ opportunities to learn; the Traditional-group ratings were significantly lower than the ratings for either the Support or the Integrating Mathematics Assessment group. It is important to acknowledge that we can make no inferences about the potential of improved curriculum without any professional support, because both the Support and the IMA teachers were participating in a professional development program. We have no evidence of the opportunities to learn that may emerge when teachers implement problem-solving curricula without any professional support.

The findings for Numerics indicated that use of reform curricula that emphasize graphic representations may reduce students’ opportunities to engage with numeric
representations; the ratings for the Support group were lower than the ratings for either the IMA or the Traditional group. The finding of a significant difference between the IMA and Support groups provided evidence that a professional development program like IMA, designed to enhance teachers’ understandings of mathematics, can contribute to greater opportunity for students to engage with numeric representations in ways that help students build understandings of concepts.

Our findings raise cautions about comparisons between traditional and reform practices in mathematics classrooms in that the patterns of comparisons among groups differed for our two indicators. One set of findings fit a more straightforward comparison between reform and traditional practice: Use of the problem-solving curriculum, compared with use of traditional textbooks, was associated with more opportunities for students to engage in discussions of the concepts underlying problem solving. However, the second set of findings revealed differences between the two reform groups: For the Support group, use of the reform curriculum, compared with use of textbooks, was associated with a lower rating for opportunity to engage with numeric representations in ways that help students build understandings of fractions concepts. The second pattern made clear that whereas improved curriculum materials can provide rich activities that support students’ mathematical investigations, in and of themselves such materials may not be sufficient enablers of instruction that affords pursuit of conceptual issues (Ball & Cohen, 1996).

**Implications and Applications**

Our work is a contribution to the repertoire of tools for estimating opportunity-to-learn, and our findings have implications for the professional practice of teachers and teacher educators.

**New measures.** Our measures supplement other methods currently being used to gather evidence of relationships between instructional practices in mathematics classrooms and students’ learning gains. Of particular note are the procedures developed by Stein and Lane (1996) and Stein et al. (1996) for sampling and coding mathematical tasks and linking those findings to student outcomes. These investigators—working much as we did when designing our rating scales—developed schemes that reflected both current research on mathematics learning and their project questions and data. When applied to print materials and classroom observation notes, the codes were designed to describe mathematics tasks, the cognitive demands and affordances of tasks, and the ways students work on tasks in the classroom. Consistent with our IMA findings, the results demonstrated the benefits of professional development (QUASAR) for students’ opportunities to engage with mathematical concepts and problem solving (e.g., multiple strategies, connected representations, oral and written explanations); furthermore, students’ learning gains were greater in classrooms in which teachers provided these opportunities. The work of these researchers provides investigators with a different set of tools, or with models of tools, for documenting the extent to which instructional practices are aligned with current understandings of what is effective. Researchers’ ultimate choices of methods will
emerge from their research questions, the sizes of their studies, the funds available for data collection and analysis, their awareness of different methods of sound technical quality, and their prior experiences and comfort with particular methods.

Access to a repertoire of tools for capturing opportunity-to-learn may benefit practitioners and preservice teachers as well as the research community. For example, when teachers are analyzing whole-class lessons, videotapes are invaluable resources, and rating scales like ours may help teachers focus their discussions. When teachers are analyzing curriculum, instructional materials and implementation field notes are useful evidence, and coding schemes (e.g., Stein & Lane, 1996) may help teachers capture relationships between curriculum tasks and emergent opportunities to learn. Future studies are needed to determine how teachers and teachers-in-training might use research measures of opportunity-to-learn to evaluate methods of instruction.

Professional practice of teachers and teacher educators. Our study findings have implications for strategies to enhance teachers’ instructional effectiveness and for classroom practice. First, our findings underscored the critical need for teacher support. Effective implementation of reform curriculum in mathematics may require substantial support for teachers’ knowledge of mathematics, children’s learning, and methods of assessment. When IMA teachers were provided opportunities to analyze the mathematical content of a curriculum unit and evidence of student learning, their students were provided greater opportunities to engage in conceptual analyses of problem solving linked to their understandings. We recognize that the IMA program was complex, and we cannot identify from this study the contributions of any specific component. Future research is needed to show how particular strategies of a professional support like IMA, as well as particular features of curriculum, influence teachers’ practices and students’ learning. Indeed, although we conceptualized the various dimensions of the IMA program—the focus on mathematics, the focus on children’s mathematics, and the focus on classroom assessment—to be supportive of one another, it may be that IMA would have been equally effective in supporting teachers’ practices with fewer components. There is also a need for cross-fertilization of studies on preservice education and studies on professional development; strategies that are appropriate for experienced teachers—for example, close analyses of specific curriculum and students’ work—may or may not be effective “jump-starts” for preservice teachers.

Second, with our findings we validated the perspective on classroom practices formulated in the NCTM Standards. Students were more likely to gain in their understandings of fractions in classrooms in which they had greater opportunities to participate in lessons that were built upon their thinking, that addressed conceptual issues in problem solving, and that used numeric representations in ways that revealed links to mathematical concepts. A curriculum unit rich in challenging problems afforded these kinds of opportunities.

Our measures captured valued dimensions of opportunity-to-learn, yet our schemes should not be interpreted as a direct “map” to instructional improvement.
Our schemes were designed to frame broad dimensions of practice and only in that sense to capture what emerges in the interactions among a teacher, the students, and the curriculum materials during a whole-class lesson. Further qualitative analyses are needed to identify the ways that specific practices emerge and the strategies and resources that teachers and students can use to construct productive teaching and learning activities.

CONCLUDING REMARK

Researchers and teacher educators are faced with two critical questions with regard to professional support and classroom practice: What kinds of support help teachers implement problem-solving curricula effectively? How do we know whether strategies for teacher support are effective?

We saw these two issues as deeply related, viewing advances in one as critical for guiding advances in the other. Thus, by implementing contrasting strategies to support teachers’ efforts to align their practices with reform frameworks, we set the stage for evaluating measures that could reveal opportunities to learn more closely aligned with reform frameworks. Reciprocally, by developing techniques to assess opportunity-to-learn keyed to reform frameworks, we set the stage for evaluating the effectiveness of our strategies for teacher support.

Our experience in designing professional development programs, documenting opportunities to learn in a range of classrooms, and studying student learning in these classrooms set us on an extended journey in the development of measures for understanding mathematics teaching and learning in the upper elementary grades. We suspect that our approach can be profitably extended to other grade levels, supporting understanding of what works in mathematics education and how to assess what works.

REFERENCES


APPENDIX

A key hierarchical linear model (HLM) that we fit to student Problem Solving posttest scores ($POSTC$) is as follows. In a Level 1, or within-class, model, we have

$$POSTC_{ij} = \beta_{0j} + \beta_{1j} \cdot (PREC_{ij} - \overline{PREC}) + \beta_{2j} \cdot (LANG_{ij} - \overline{LANG}) + r_{ij}$$

where $POSTC_{ij}$, $PREC_{ij}$, and $LANG_{ij}$ represent, respectively, the Problem Solving posttest score, the Problem Solving pretest score, and the language proficiency status for student $i$ in class $j$, and $\overline{PREC}$ and $\overline{LANG}$ represent the sample grand means for $PREC$ and $LANG$. By virtue of centering student pretest scores and language status around their grand means, $\beta_{0j}$ represents the mean Problem Solving posttest score for class $j$ adjusted for differences among classrooms in pretest scores and language status. Thus, for example, if pretest scores in class $j$ are on average lower than the grand mean ($\overline{PREC}$), the mean posttest score for class $j$ ($\beta_{0j}$) will be adjusted upwards (see Bryk & Raudenbush, 1992, pp. 19, 97). The $r_{ij}$ are within-class student residuals assumed normally distributed with mean 0 and variance $\sigma^2$.

In a Level 2, or between-class, model, differences between classes in their adjusted means are modeled as a function of $OTLC$ and $OTLN$ (our orthogonal OTL variables):

$$\beta_{0j} = \gamma_{00} + \gamma_{01} \cdot OTLC_j + \gamma_{02} \cdot OTLN_j + U_{0j}$$
$$\beta_{1j} = \gamma_{10}$$
$$\beta_{2j} = \gamma_{20}$$

where $OTLC_j$ and $OTLN_j$ represent, respectively, the OTL conceptual and OTL procedural scores for class $j$. Key parameters in the Level 2 model are $\gamma_{01}$ and $\gamma_{02}$. $\gamma_{01}$ captures the expected amount of change in class posttest performance when $OTLC$
increases 1 unit, adjusting for differences among classes in pretest performance and language status. Similarly, $\gamma_{02}$ captures the expected amount of change in $\beta_{0j}$ when OTLN increases 1 unit. The $U_{0j}$ are random effects assumed normally distributed with mean 0 and variance $\tau_{00}$.

As reported in the text of the article, when we fit this model to the data, we obtain an estimate for $\gamma_{01}$ of 0.87 points with standard error 0.21. Without any covariate adjustment (i.e., when we refit the above HLM omitting $PREC_{ij}$ and $LANG_{ij}$ from the Level 1 model), we obtain a slightly larger estimate for $\gamma_{01}$.

We also refit the HLM defined above with class $PREC$ and $LANG$ means included in the Level 2 model as follows:

$$
\beta_{0j} = \gamma_{00} + \gamma_{01} \text{OTLC}_j + \gamma_{02} \text{OTLN}_j + \gamma_{03} \text{PREC}_j + \gamma_{04} \text{LANG}_j + U_{0j}
$$

$$
\beta_{1j} = \gamma_{10}
$$

$$
\beta_{2j} = \gamma_{20}
$$

This Level 2 model combined with the Level 1 model specified above provides us with estimates of the effects of OTL, adjusting for differences in intake among classes in pretest scores and language background as before. Additionally, it provides an adjustment for possible contextual effects (e.g., possible additional benefits in posttest performance that may result, for example, when high-ability students interact with one another) (see Bryk & Raudenbush, 1992, pp. 121–122). This model also yields an estimate for $\gamma_{01}$ (i.e., 0.82) that is statistically significant.

Estimates of the effects of OTLN were not statistically significant in any of the above analyses. A similar modeling strategy was used in analyzing procedural posttest scores, and none of the resulting estimates of the effects of OTLC or OTLN were statistically significant.

When we employ normality assumptions in statistical modeling, outlying cases can have a large effect on parameter estimates and their standard errors. In the case of HLMs, outlying classrooms (e.g., classrooms in which achievement is unusually low or high) can influence estimates and standard errors for parameters of interest in Level 2 models. To check for possible outlying classrooms in our analyses, we constructed plots of Mahalanobis Distances as described in Bryk and Raudenbush (1992, pp. 218–220). No outlying classrooms were detected.
Opportunities to Learn Fractions in Elementary Mathematics Classrooms
Maryl Gearhart; Geoffrey B. Saxe; Michael Seltzer; Jonah Schlackman; Cynthia Carter Ching; Na'ilah Nasir; Randy Fall; Tom Bennett; Steven Rhine; Tine F. Sloan
Stable URL: [http://links.jstor.org/sici?sici=0021-8251%28199905%2930%3A3%3C286%3AOTLFIE%3E2.0.CO%3B2-T](http://links.jstor.org/sici?sici=0021-8251%28199905%2930%3A3%3C286%3AOTLFIE%3E2.0.CO%3B2-T)

This article references the following linked citations. If you are trying to access articles from an off-campus location, you may be required to first logon via your library web site to access JSTOR. Please visit your library's website or contact a librarian to learn about options for remote access to JSTOR.

**References**

*The Mathematical Understandings That Prospective Teachers Bring to Teacher Education*
Deborah Loewenberg Ball
Stable URL: [http://links.jstor.org/sici?sici=0013-5984%28199003%2990%3A4%3C449%3AMUTPT%3E2.0.CO%3B2-M](http://links.jstor.org/sici?sici=0013-5984%28199003%2990%3A4%3C449%3AMUTPT%3E2.0.CO%3B2-M)

*Prospective Elementary and Secondary Teachers' Understanding of Division*
Deborah Loewenberg Ball
Stable URL: [http://links.jstor.org/sici?sici=0021-8251%28199003%2921%3A2%3C132%3AEASTU%3E2.0.CO%3B2-5](http://links.jstor.org/sici?sici=0021-8251%28199003%2921%3A2%3C132%3AEASTU%3E2.0.CO%3B2-5)

*With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics*
Deborah Loewenberg Ball
Stable URL: [http://links.jstor.org/sici?sici=0013-5984%28199303%2993%3A4%3C373%3AWAEOTM%3E2.0.CO%3B2-F](http://links.jstor.org/sici?sici=0013-5984%28199303%2993%3A4%3C373%3AWAEOTM%3E2.0.CO%3B2-F)
Reform by the Book: What Is: Or Might Be: The Role of Curriculum Materials in Teacher Learning and Instructional Reform?
Deborah Loewenberg Ball; David K. Cohen
Stable URL:
http://links.jstor.org/sici?sici=0013-189X%28199612%2925%3A9%3C6%3ARBTBWI%3E2.0.CO%3B2-M

Assessment of a Problem-Centered Second-Grade Mathematics Project
Paul Cobb; Terry Wood; Erna Yackel; John Nicholls; Grayson Wheatley; Beatriz Trigatti; Marcella Perlwitz
Stable URL:
http://links.jstor.org/sici?sici=0021-8251%28199101%2922%3A1%3C3%3AAOAPSM%3E2.0.CO%3B2-%23

A Revolution in One Classroom: The Case of Mrs. Oublier
David K. Cohen
Stable URL:
http://links.jstor.org/sici?sici=0162-3737%28199023%2912%3A3%3C311%3AAARIOCT%3E2.0.CO%3B2-9

The Effects of Instructing Teachers about Good Teaching on the Mathematics Achievement of Fourth Grade Students
Howard Ebmeier; Thomas L. Good
Stable URL:
http://links.jstor.org/sici?sici=0002-8312%28197924%2916%3A1%3C1%3ATEOITA%3E2.0.CO%3B2-T

A Longitudinal Study of Learning to Use Children's Thinking in Mathematics Instruction
Elizabeth Fennema; Thomas P. Carpenter; Megan L. Franke; Linda Levi; Victoria R. Jacobs; Susan B. Empson
Stable URL:
http://links.jstor.org/sici?sici=0021-8251%28199607%2927%3A4%3C403%3AAALSOLT%3E2.0.CO%3B2-8
Learning to Teach Mathematics in the Context of Systemic Reform
S. G. Grant; Penelope L. Peterson; Angela Shojgreen-Downer
Stable URL:
http://links.jstor.org/sici?sici=0002-8312%2819962%2933%3A2%3C509%3ALTMIT%3E2.0.CO%3B2-%23

Who Is Minding the Mathematics Content? A Case Study of a Fifth-Grade Teacher
Ruth M. Heaton
Stable URL:
http://links.jstor.org/sici?sici=0013-5984%28199211%2993%3A2%3C153%3AWIMTMC%3E2.0.CO%3B2-N

Instructional Tasks, Classroom Discourse, and Students' Learning in Second-Grade Arithmetic
James Hiebert; Diana Wearne
Stable URL:
http://links.jstor.org/sici?sici=0002-8312%2819932%2930%3A2%3C393%3AITCDAS%3E2.0.CO%3B2-D

Teachers' Professional Development in a Climate of Educational Reform
Judith Warren Little
Stable URL:
http://links.jstor.org/sici?sici=0162-3737%2819932%2915%3A2%3C129%3AETPDIAC%3E2.0.CO%3B2-I

Teachers' and Students' Cognitional Knowledge for Classroom Teaching and Learning
Penelope L. Peterson
Stable URL:
http://links.jstor.org/sici?sici=0013-189X%28198806%2917%3A5%3C5%3ATASCKF%3E2.0.CO%3B2-L

Are Changes in Views about Mathematics Teaching Sufficient? The Case of a Fifth-Grade Teacher
Richard S. Prawat
Stable URL:
http://links.jstor.org/sici?sici=0013-5984%28199211%2993%3A2%3C195%3AACIVAM%3E2.0.CO%3B2-T
Teachers' Beliefs about Teaching and Learning: A Constructivist Perspective
Richard S. Prawat
Stable URL:
http://links.jstor.org/sici?sici=0195-6744%28199205%29100%3A3C354%3ATBATAL%3E2.0.CO%3B2-L

Teaching the "Hows" of Mathematics for Everyday Life: A Case Study of a Fifth-Grade Teacher
Ralph T. Putnam
Stable URL:
http://links.jstor.org/sici?sici=0013-5984%28199211%2993%3A2%3C163%3ATT%22OMF%3E2.0.CO%3B2-J

Significant and Worthwhile Change in Teaching Practice
Virginia Richardson
Stable URL:
http://links.jstor.org/sici?sici=0013-189X%28199010%2919%3A7%3C10%3ASAWCIT%3E2.0.CO%3B2-7

Relations between Classroom Practices and Student Learning in the Domain of Fractions
Geoffrey B. Saxe; Maryl Gearhart; Michael Seltzer
Stable URL:
http://links.jstor.org/sici?sici=0737-0008%281999%2917%3A1%3C1%3ARBCPAS%3E2.0.CO%3B2-T

Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms
Mary Kay Stein; Barbara W. Grover; Marjorie Henningsen
Stable URL:
http://links.jstor.org/sici?sici=0021-9624%28199622%2933%3A2%3C455%3ABSCFMT%3E2.0.CO%3B2-%23
The Value (And Convergence) of Practices Suggested by Motivation Research and Promoted by Mathematics Education Reformers
Deborah Stipek; Julie M. Salmon; Karen B. Givvin; Elham Kazemi; Geoffrey Saxe; Valanne L. MacGyvers
Stable URL:
http://links.jstor.org/sici?sici=0021-8251%28199807%2929%3A4%3C465%3ATV%28COP%3E2.0.CO%3B2-5