


Our number system is a legacy of our culture. The study of how mothers teach their children about number provides important insights into the way in which children come to intertwave their own developing understandings with achievements that have occurred in our culture's social history.

The Social Organization of Early Number Development

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Research in cognitive development is generally motivated by the concern to understand the nature of developmental shifts in children's conceptual understandings and logical operations. To pursue this research interest, investigators have often studied children removed from the everyday social contexts in which they use their conceptual skills, and they have interviewed and observed children as they solved problems without the support and collaboration of others. While divorcing the investigation of children's conceptual development from the social contexts in which it occurs permits the researcher greater access to the child's reasoning processes, it removes from observation the ways in which the development of children's reasoning is supported and informed by interactions with others.

The writings of Vygotsky (1962, 1978) and recent elaborations of his works (for example, Wertsch, 1979) address the social network of meanings, activities and historical achievements within which the individual operates and

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learns. Using such constructs as the zone of proximal development, Vygotsky argues that an analysis of the social organization of a child's problem-solving efforts is essential to an understanding of cognitive development and its cultural origins. Vygotsky's approach has provided us with a framework for investigating the social roots of one domain of cognitive development, children's numerical cognition.

The development of number is a particularly fruitful domain for the investigation of developmental relations between culture and cognition. Number systems are evolving cultural constructions. This is apparent both in the wide cultural diversity of number systems (Saxe and Posner, 1982) and in the remarkable history of our own number system and procedures for calculation (Menninger, 1969). In focusing on aspects of the social organization of children's early number development, we gain access to a process whereby an evolving cultural construction—the number system—is communicated to children and children transform and incorporate that construction into the fabric of their own cognitive activities. In our research, we have been examining that process by observing how mothers teach their children to solve a counting problem.

Our analysis of adult-child interactions is set within a general model of cognitive development. It is our view that children's novel cognitive constructions result from the dynamic interplay between their elaboration of problem-solving goals and coherent means to achieve those goals. As children identify new goals, they attempt to elaborate novel cognitive means, including conceptual structures, symbolic vehicles, and problem-solving strategies, in order to achieve those goals. These cognitive constructions in turn provide a new framework within which individuals attempt to identify new goals. The aim of our research on mother-child interactions is to explore whether and how mothers participate in the child's elaboration of problem-solving goals and problem-solving means.

**Research Plan**

In order to study the social organization of goals during mother-child teaching interactions about number, we videotaped mothers and their two-and-one-half- to five-year-old children as the mothers attempted to teach their children a number reproduction game. We also interviewed children individually to obtain a characterization of children's unassisted performances on the number reproduction and other related tasks. For the interaction session, we instructed mothers that the goal of the number reproduction game was to get the same number of pennies (from an available set of fifteen) as there were Cookie Monsters (pictures of the puppet from the "Sesame Street" television show) on the model board, and we encouraged mothers to organize the interaction in whatever way they felt would encourage learning and understanding in their child. We asked the mothers to keep the pennies five to six feet away from the model and to have their children bring the pennies back in a cup.

These instructions were designed to discourage the mothers from organizing local task completion strategies, such as pairing pennies with cookie monsters one by one, strategies that radically simplify the goal of numerical reproduction. Mothers helped their children to complete the task four times for model set sizes of three, four, nine, and ten, in that order. We thus varied task difficulty according to the number of Cookie Monster pictures in the model.

To understand the goal structure that emerges during the adult-child teaching interactions, we found it necessary to develop a method of study in which we produced a coordinated set of analyses of three aspects of the numerical activity. The method entailed an analysis of the goal structure of the activity as it was understood by the mother (or practiced in "culture"), a developmental analysis of the goal structure that children imposed on the activity, and an analysis of how the adult participated in the child's construction of the goals in the activity. In the discussion that follows, we show that each of these analyses is a necessary complement of the others, and we show that together they lead to new insights about how children's developing numerical understandings are jointly rooted in their own constructive activities and in their social interactions with others.

**Analysis of the Goal Structure of the Number Reproduction Task**

To understand the functional requirements of any task—the work that the subject needs to accomplish in order to solve the task—an analysis of the goal structure that leads to task solution is necessary. The counting game that we asked mothers to teach to their children is similar to many everyday counting activities. To reproduce a given quantity of objects, as our task requires, an individual must accomplish a hierarchy of goals and subgoals. The superordinate goal is to produce an accurate numerical copy. In order to accomplish this, the subject must first accomplish a subgoal: produce an accurate estimate of the model. One means of accomplishing this subgoal is by counting the model. In order to count, further subgoals must be elaborated and accomplished, such as applying the sequence of number words in one-to-one correspondence with the target elements. Once the goals pertaining to the model have been achieved, a similar set of subgoals must be constructed to obtain an accurate number of objects from the available set. Finally, the individual can choose to check his or her accuracy by either recounting the sets or by establishing a one-to-one correspondence between the two sets.

Although the description of the goal structure just presented corresponds to the procedures that we as adults would use to produce a solution to the numerical reproduction task, it does not correspond to the organization of the child's activity as he or she proceeds to solve the task. In order to understand how adults can influence and elaborate children's goals, it is first necessary to understand children's goals during their solution of the task.
A Developmental Analysis of Children’s Goals

An analysis of children’s unassisted performances on the numerical reproduction task reveals that younger children do not simply make errors in their solution of the task but that they conceptualize the task quite differently than adults do. Table 1 contains a summary description of developmental shifts in children’s unassisted performance on numerical reproduction tasks and what we infer to be general features of children’s goals associated with these behaviors (see Saxe 1977, 1979).

Young children (Level 1) who are presented with the task often act as if they have two distinct and fluctuating goals during this activity. One goal is to get some or all the elements in the available set. If, during their activity, these children are asked whether counting the model would help them, some (Level 1A) seem to construct another goal: to produce a count of one set or of both sets continuously as if they were one. Slightly more advanced children (Level 1B) focus on producing separate counts of the sets but do not use the information obtained to relate the copy to the model. It is important to note that when children at Levels 1A or 1B identify counting as a goal of the activity, they seem to do so only with regard to the production of a count of a single array. Thus, if they produce two counts (Level 1B), they treat their counts separately, and they do not compare the values that they produce. Moreover, children at these levels often do not treat the last number word of their count as having a cardinal value. For instance, if a child at either sublevel counts a set and is then asked how many items there are in the set, the child is likely either to recount the set or to offer the last several number words of the count as a reply.

At Level 2, children’s goals seem to shift. Now, children exhibit double array goals: They produce separate and distinct numerical representations of the model and copy, which they compare. Nonetheless, their solution strategy at Level 2A remains different from that of adults. Children at Level 2A count the model and available set without a clearly articulated overall plan for the task. Then, through a process of recounts and successive additions and subtractions, they attempt to equalize the model and their copy. At Level 2B, children produce systematic counts and an accurate reproduction of the model.

A Framework for Understanding the Emergent Goal Structure of the Activity During Mother-Child Interaction

The actual goal structure of the activity as it emerged over the course of mother-child interactions was analyzed by constructing coding schemes that were guided both by a logical analysis of the goal structure of the number reproduction activity as communicated to the mothers and by developmental analysis of the child’s shifting goals.

We suspected that adults interweave their instruction with children’s ongoing problem-solving activities by adjusting the goal structure of tasks to children’s level of functioning. For instance, in presenting the number reproduction task to children, mothers can define or help their children to accomplish goals at any one of many levels of task structure. Table 2 contains the hierarchical description of the goals and subgoals of the task. Mothers can offer directives during the interaction at any of these levels.

At the most general goal structure levels, the mother presents the goal structure of the entire task without specifying any of the subgoals (directives 1 and 2). At each of these levels of description, the mother provides a double array goal—the child must produce and compare numerical representations of the model and the copy. At the next level (directive 3), the mother guides the child to a specific subgoal by directing the child to get a specific number of pennies from the available set. This form of directive is transitional between a double array and a single array goal specification of the task in that the mother directs her child to produce a specific representation of the available set of pennies (single array goal), but she does so in the context of achieving a representation of the model set. The remaining directives entail increasing specificity of how to achieve a numerical representation of the model set, each referring to a single array goal or to some aspect of a single array goal. For instance, at the fourth level, the mother asks the child for a numerical representation of the model without providing information on how to accomplish this. At the fifth level, the mother specifies not only the subgoal concerning the need for a representation but also the further subgoal concerning the need to count to achieve the representation. The remaining levels each represent more specific directives concerning how to achieve an accurate count.

Table 1. Developmental Analysis of Children’s Goals on Numerical Reproduction Task

<table>
<thead>
<tr>
<th>Level</th>
<th>Child’s Behavior</th>
<th>Inferred Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>Child brings all of available set. “Would counting help?” Child counts only own copy.</td>
<td>Single array goals</td>
</tr>
<tr>
<td>1B</td>
<td>Same as 1A, but child counts model and copy and produces no subsequent modifications.</td>
<td>Single array goals</td>
</tr>
<tr>
<td>2A</td>
<td>Child equalizes model and copy through successive counts, additions, and subtractions.</td>
<td>Double array goals</td>
</tr>
<tr>
<td>2B</td>
<td>Systematic reproduction using counting.</td>
<td>Double array goals</td>
</tr>
</tbody>
</table>

Analyses of the Interactions

On the basis of the children’s unassisted performances, we divided them into two groups, a low-ability group and a high-ability group. We inferred that these groups would tend to apply single and double array goals respectively to
Table 2. A Hierarchical Ordering of Maternal Directives for the Numerical Reproduction Task

<table>
<thead>
<tr>
<th>Maternal Directive</th>
<th>Example</th>
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<tbody>
<tr>
<td>Directives Pertaining to the Goal Structure of the Entire Task</td>
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<tr>
<td>1. Mother provides the superordinate goal of the entire task.</td>
<td>&quot;Get just the same number of pennies as Cookie Monsters.&quot;</td>
</tr>
<tr>
<td>2. Mother provides the superordinate goal of the entire task after a representation of the model has been accomplished.</td>
<td>&quot;Get just the same number of pennies as Cookie Monsters.&quot;</td>
</tr>
<tr>
<td>Directives Linking a Representation of the Model with a Production of a Copy</td>
<td></td>
</tr>
<tr>
<td>3. Mother directs the child to obtain a specified number from the available set.</td>
<td>&quot;Go get nine pennies for the Cookie Monsters.&quot;</td>
</tr>
<tr>
<td>Directives to Produce a Representation of the Model</td>
<td></td>
</tr>
<tr>
<td>4. Mother directs child to produce a representation of the model without specifying how to do so.</td>
<td>&quot;How many Cookie Monsters are there?&quot;</td>
</tr>
<tr>
<td>5. Mother directs child to produce a representation of the model and specifies a procedure whereby this can be accomplished.</td>
<td>&quot;Count the Cookie Monsters.&quot;</td>
</tr>
<tr>
<td>6-10. Mother provides increasing assistance on some aspect of the child’s counting activity with each successive level.</td>
<td>Mother counts as child points to each Cookie Monster.</td>
</tr>
<tr>
<td>Directives Providing a Representation of the Model</td>
<td></td>
</tr>
<tr>
<td>11. Mother provides a cardinal representation of the model for the child.</td>
<td>&quot;There are three Cookie Monsters.&quot;</td>
</tr>
</tbody>
</table>

the reproduction tasks during the interactions. We then conducted a number of analyses on the interactions using the scheme presented in Table 2 to determine whether and in what way mothers adjusted the organization of the task to the child’s understanding of the task’s goal structure.

Task Introductions. First, we asked whether mothers introduced the task differently to children of different ability levels of numerical competence. For this analysis, we examined introductions only to set sizes three and nine, since many mothers organized set sizes four and ten as problems of addition to set sizes three and nine, not as independent trials.

We found that mothers were making adjustments appropriate to the ability levels of their child. The mothers of low-ability children introduced the task differently from the mothers of high-ability children. Most mothers of low-ability children began with a single array subgoal request (for example, a Level 4 “How many Cookie Monsters are there?” or a Level 5 “Count the Cookie Monsters”). A few mothers of low-ability children formulated a superordinate double array goal, but they used it only as a context for a more specific single array subgoal request. Here is an example of such a strategy:

Example 1. Low-Ability Child; Set Size: 3

Mother: Are you ready to learn a game? (Mother leans over to look in child’s face. Mother gives cup to child.) Okay [What] we’re going to do is we’re going to count the Cookie Monsters. Okay? (Mother pushes set size three Cookie Monster boards towards child while pointing to each monster.) And, then I want you to go over to the pile of pennies over there and put the same number of pennies in the cup... so all the Cookie Monsters have one. (Superordinate as Context)

Child: Okay.
Mother: Okay? So, should we count the Cookie Monsters? (Level 5 directive)

A few mothers of low-ability subjects immediately transformed the entire goal structure of the task and presented a simplified goal structure (not coded within the levels analysis); for example, “I want you to get a cookie [a penny] and put it in the cup to give it to the Cookie Monster.” Such a simplification redefined the task from a numerical one to one that involved non-numerical correspondences. Thus, the task structure presented to these children was neither one that entailed double nor single array goals. Instead, the goal merely entailed getting elements of a set. We find these adjustments all the more significant in light of our attempts to prevent such radical simplifications in our initial instructions to the mothers.

No mother of a high-ability child ever immediately simplified the goal structure of the task; nor did any of these mothers specify the means for achieving a representation of the model, that is, counting (Level 5). The typical introduction for mothers of high-ability children was, “How many Cookie Monsters are there?” (Level 4), a request intended to help focus the child on the need to achieve a numerical representation of a single array (the model set of Cookie Monsters) without specifying the means for doing so. Some mothers of high-ability children specified just the superordinate goal (for example, “You have to get the same number of pennies as there are Cookie Monsters and put them in the cup” [Level 1]). In contrast, mothers of low-ability children never specified just a superordinate goal.

Mothers tended to introduce set size nine numerical reproductions differently from set size three. Some mothers in both high- and low-ability groups adapted their task introductions to set size nine by increasing their assistance and initiating the task with a more subordinate directive. In addition, we found another form of adaptation for the mothers of high-ability children. The trial for set size nine was always preceded by trials for set sizes three and four. Some of the mothers of high-ability children made use of these previous interactions by introducing the task with a request that the child merely
“Do this one now”—an “empty” task marker that displayed the mother's belief that after two trials her child now shared with her an understanding of the task goals. No mother of a low-ability child ever introduced the task with only a marker of this nature.

Formulations of Superordinate Goals. The preceding analysis indicates that mothers of high-ability and low-ability children differed in the degree to which they specified the task structure in their introductions. Mothers of low-ability children tended to structure the subgoals for their children to a much greater extent than did mothers of high-ability children. In a related analysis, we examined whether mothers ever formulated the superordinate goal of numerical reproduction (Level 1 or Level 2 in Table 2) at any point in the model phase (when the child was to determine the number of Cookie Monsters) or in the initiation of the available set phase (when the child went to get the same number of pennies).

In set size three, most mothers of both low-ability and high-ability children formulated the superordinate task goal at some point before the child gathered pennies from the available set. In set size nine, however, most mothers of low-ability subjects did not formulate the superordinate goal, while most mothers of high-ability subjects were still likely to formulate the superordinate goal. It is likely that, by set size nine, the mothers of low-ability subjects had learned from repeated task trials that the superordinate double array goal was not understood by their children and thus that formulation of it was not useful in supporting the child's task activity.

Median Assistance Levels. In order to obtain an index of the extent to which the mothers themselves structured the subgoals of the task for their children, we next calculated a median score for each mother's goal directives in the model phase for each set size. Consistent with the analyses of task introductions and superordinate goal formulations, we found that mothers of high-ability children gave their children less assistance in constructing the subgoals of the task than did mothers of low-ability children. We also found that task complexity influenced the teaching strategies of mothers of both high-ability and low-ability children. As task complexity increased, both groups of mothers shifted to more subordinate goal directives. Examples 2 and 3 illustrate the way in which the mother of a high-ability child adjusts the goal structure of the task as a function of the model's set size. In this case, the adjustment occurs in the task introduction.

Example 2. High-Ability Child; Set Size 3; Median Assistance: 3.0
Mother: Look at this board. Now, can you tell me how many Cookie Monsters are on there? (Level 4 directive)
Child: Okay. One, two, three. (Child touches each Cookie Monster.)
Mother: Okay. Now, the trick is this. We have to—you have to get the same amount of pennies and put them in the cup that are on here. (Mother points to cup and touches board. Child goes to penny pile.) (Level 2 directive)

Example 3. High-Ability Child; Set Size: 9; Median Assistance: .5
Mother: Close your eyes. (Mother gets set size nine board. Child opens eyes and gasps.) Wow! Now, can you count? (Level 5 directive)
Child: One, two, three, four, five, six, seven, eight, nine. (Child touches each Cookie Monster in turn.)
Mother: Okay. How many Cookie Monsters are there? (Uptake on last numeral)
Child: Nine. (Child “sings,” sweeping hand around board.)
Mother: Now we have to put the same amount in the cup that are here. (Level 2 directive)

Mothers' Shifts in Goal Directives Within Set Size. Not only do mothers adjust their organization of the task as a function of the child's ability level and task difficulty, but the social organization of the task is dynamic and shifts during an interaction as a function of whether the child achieves an accurate count. We found that mothers generally shifted to a goal directive subordinate to their previous one after the child produced an inaccurate count and that mothers generally shifted to a superordinate goal directive after the child produced an accurate count. These trends occurred regardless of task difficulty and the child's ability level. Example 4 illustrates a shift to a subordinate goal following an incorrect count (Level 4 to Level 10 assistance) and a shift to a more superordinate goal following an accurate count (Level 10 to Level 3 assistance).

Example 4. High-Ability Child; Set Size: 9; Median Assistance: 4.0
Mother: Here's one that's much harder. (Mother puts out set size nine board.) How many Cookie Monsters do we have here? (Level 4 directive)
Child: One, two, three, four, five, six, seven, eight, nine, ten. (After two, child points either miss or repeat Cookie Monsters.)
Mother: Let me help you count them, okay? (Mother and child count and point in unison until five, whereupon child mistakenly says seven.) Forgot, you forgot about five. (Mother and child count and point in unison from five to nine.) (Level 10 assistance) Golly, that's a lot. Can you count nine pennies and put them in the cup? (Level 3 directive)

Mothers' Uptake on Children's Accurate Counts. The use of the last number word of the count to represent the quantity in the model array presents special difficulty for children, and the analysis of developmental shifts in this understanding has received considerable attention in the research literature on children's early cognitive development (Gelman and Gallistel, 1978; Gelman and Meck, in press; Schaeffer and others, 1974). To assess how mothers signal the special utility of the last number word, we focused on what mothers said following the child's first accurate count as a function of the child's ability level. For instance, once the child completed a count, some mothers repeated the
number word; others asked the child how many there were. We found that
mothers of low-ability children provided an uptake on the last number word of
the child’s count more often than did mothers of high-ability children. In addition,
the frequency of mothers’ uptakes tended to increase with task difficulty
for high-ability children.

Summary and Conclusions

In this chapter, we have provided coordinated analyses of three aspects
of the social context of children’s developing conceptual understandings: an
analysis of children’s developing operations within a knowledge domain, a
functional analysis of the cultural task context in which these operations are
deployed, and an analysis of the way in which other people can bridge and
adapt the cultural definition of the task to the child’s developing operations. We
believe that the insights gained from this type of analytic approach are critical
in understanding the way in which young children come to make the historical
achievements of culture a part of their own problem-solving activities.

At the beginning of this chapter, we argued that very young children
who engage in the number reproduction activity often impose the goal of pro-
ducing a count of a single array (Levels 1A and 1B) on the nominal task and
that their means of accomplishing the count are typically not well developed.
The results for our low-ability subjects indicate that their mothers provided
assistance appropriate to their developing numerical operations. For instance,
these mothers were more likely to initiate the task with specific directives, such
as, “Count the Cookie Monsters.” Moreover, as the greater median assistance
levels reveal, these mothers often assisted their children’s model set counts by
modeling or directing very low-level subgoals, such as the repetition of number
words in the correct sequence or the assignment of number words in one-to-
one correspondence with objects (Levels 6 through 11). In their uptakes to
accurate counts, these mothers also attempted to highlight to their children
that the last number word recited could be used as a summary description
(cardinal representation) of the entire array. Not surprisingly, these mothers
were less likely than the mothers of high-ability children to formulate only the
overall double array goal structure of the task, and they were less likely than
the mothers of high-ability children ever to formulate it. Thus, the mothers
themselves more often took responsibility for relating the numerical value of
the model to that of the copy. We interpret these mother-child interactions as a
context for the low-ability child to generate a system of understanding and
symbolization that is commensurate with the child’s definition of the goal
structure of the task: the representation of single arrays by means of our con-
ventional numeration system.

Older preschool children begin to construe the task as having a double
array goal structure. They attempt to produce numerical representations of
the model using the means that they have developed to achieve single array
goals. This generally entails counting the model and the available set and
then, by a trial-and-error process, equalizing them through a succession of
counts, additions and subtractions, and recounts (Level 2A). Gradually, these
older children structure increasingly systematic solution strategies so that they
organize their counting to achieve a precise copy (Level 2B). The results for
high-ability children revealed that mothers provided directives that supported
the child’s construction of these systematic solution strategies. Mothers of
high-ability children were more likely to introduce the task at superordinate
goal levels, and they were more likely to formulate the goal structure of the
entire task when using large set sizes. These mothers provided less assistance
in the model phase, and they were less likely than mothers of low-ability chil-
dren to provide uptake on the last number word of their children’s counts.
Thus, they were more likely to relinquish responsibility to the child for relat-
ing the numerical value of the model to that of the copy.

In addition to these group differences, we found that all mothers con-
tinually adjusted the goal structure of the task during the activity itself. So,
while children of low ability succeeded less often than did children of high abili-
ty, their mothers, like the mothers of high-ability children, tended to shift to
more superordinate goal directives when the children did succeed. Similarly,
when children of both ability levels had difficulty, mothers shifted to more sub-
ordinate goal levels.

The analyses that we have presented in this chapter indicate that the
goal structure of numerical activities as they occur in social interactions is an
emergent phenomenon: Located neither in the head of the mother nor in that
of the child, this goal structure is negotiated in the interaction itself. Thus, the
emergent goal structure simultaneously involves the child’s understandings
and the historical achievements of culture as communicated by the mother.
We argued that children construct means to achieve these socially negotiated
goals. For a young child, this can entail the imitation of the number string in
the same sequence as the mother articulates it. For an older child, it can entail
discovering the importance of systematically counting both the model and the
copy. As children generate coherent means to achieve these socially negoti-
tated goals, they create for themselves a system of representation that reflects
achievements that have been generated in our culture’s social history.

References

Young children rapidly develop into socialized participants in their culture through a finely tuned combination of infants’ skills and the guidance of more experienced people.

Interaction with Babies as Guidance in Development

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Newborn infants are quite ignorant regarding the workings of the society into which they are born. By age three, however, children are socialized participants in their culture. It is the thesis of this chapter that the rapid development of babies into socialized participants in society is accomplished through a finely tuned combination of the infant's skills and the guidance of more experienced people. First, we discuss some characteristics of infants that seem suited for quickly picking up great amounts of information about their new environment. Then, we describe some characteristics of adult-infant interaction that we regard as well adapted to the gradual immersion of infants in the skills and beliefs of the society. (See also Rogoff, in press.) Our argument that infant skills and adult-infant interactional strategies together produce development is

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